INCLUSION IN TEXAS

Instructional Routines for Mathematics Intervention

The purpose of these mathematics instructional routines is to provide educators with materials to use when providing intervention to students who experience difficulty with mathematics. The routines address content included in the grades 2-8 Texas Essential Knowledge and Skills (TEKS). There are 23 modules that include routines and examples – each focused on different mathematical content. Each of the 23 modules include vocabulary cards and problem sets to use during instruction. These materials are intended to be implemented explicitly with the aim of improving mathematics outcomes for students.

TEA
Texas Education Agency

Instructional Routines for Mathematics Intervention

MODULE 11

Multiplication of Whole Numbers

Module 11: Multiplication of Whole Numbers
Mathematics Routines

A. Important Vocabulary with Definitions

Term	Definition
algorithm	A procedure or description of steps that can be used to solve a problem.
area	The number of square units that covers a closed figure.
array	A set of objects, pictures, or numbers arranged in columns and rows.
commutative property of multiplication	Two factors can be multiplied in any order.
computation	The action used to solve a problem.
equal groups	Groups with the same number of objects or items in each group.
equal sign	The symbol that tells you that two sides of an equation are the same, balanced, or equal.
factor	A number that you multiply with another number to get the product.
hundreds column	The column with digits in the hundreds place.
multiply/multiplication	The process of adding a number to itself a number of times.
multiplication sign	The symbol that tells you to multiply.
ones column	The column with digits in the ones place.
partial products	The product of parts of each factor.
product	The result of multiplying two or more factors.
regroup/trade/exchange	The process of exchanging 10 ones for 1 ten, 10 tens for 1 hundred, 10 hundreds for 1 thousand, etc.
tens column	The column with digits in the tens place.

B. Background Information

Background Information:

If your focus is on the conceptual understanding of multiplication, see *Module 10: Concepts of Multiplication*. This module, *Module 11*, focuses on computation with multiplication of whole numbers. As you focus on computation, continue to emphasize multiplication as equal groups and multiplication as comparison because students will see these concepts within word problems.

For learning computation with multiplication, we recommend presenting problems vertically. Some students may require explicit instruction on translating a horizontal problem (e.g., 12 × 27) to the vertical presentation (see below). Depending upon the algorithm, leave enough space above or below the problem for students to complete their written work.

Every student should develop efficiency with a multiplication computation strategy. In the following sections, we provide examples of (1) multiplication with traditional algorithm, (2) multiplication with partial products algorithm, and (3) multiplication with array (or area model). Teachers should help students develop competency with at least one algorithm.

Computation with Multiplication

14 ← factor
× 32 ← factor
448 ← product

C. Routines and Examples

(1) Multiplication with Traditional Algorithm

Routine

Materials:

- Module 11 Problem Sets
- Module 11 Vocabulary Cards
  - If necessary, review Vocabulary Cards before teaching
- A hands-on tool or manipulative like Base-10 blocks or unifix cubes
  - Note that drawings can be used alongside or instead of manipulatives

2-DIGIT × 1-DIGIT: ROUTINE WITH MANIPULATIVES
(Only use manipulatives with simpler problems)

Teacher	**Let's work on multiplication. What does it mean to multiply?**
Students	To make equal groups or to compare.
Teacher	**Multiplication means to make equal groups or to compare. Look at this problem.** (Show problem.)
Teacher	**First, I see a multiplication sign** (point). **The multiplication sign tells us to multiply. What does the multiplication sign mean?**
Students	To multiply.
Teacher	**Let's do this problem with Base-10 blocks.** (Move Base-10 blocks to workspace.)

Teacher	**With our Base-10 blocks, each cube represents one thousand. What do the cubes represent?**	
Students	Thousands.	
Teacher	**The flats represent hundreds. What do the flats represent?**	
Students	Hundreds.	
Teacher	**The rods represent tens. What do the rods represent?**	
Students	Tens.	
Teacher	**With our Base-10 blocks, the units represent ones. What do the units represent?**	
Students	Ones.	
Teacher	**Our first factor is __. What's our first factor?**	
Students	__.	
Teacher	**Our second factor is __. What's our second factor?**	
Students	__.	
Teacher	**Let's solve this problem using multiplication as equal groups. What does equal groups mean?**	
Students	We have groups with an equal number in each group.	
Teacher	**In this problem, we have __ groups of __. What do we have?**	
Students	__ groups of __.	
Teacher	**If we want to use the Base-10 blocks, I want to use the commutative property. The commutative property says that, in multiplication, the order of the factors does not matter. We could multiply __** (first factor) **times __** (second factor) **or __** (second factor) **times __** (first factor). **The product will be the same. What's the commutative property?**	
Students	In multiplication, the order of factors does not matter.	
Teacher	**So, in this problem. I want to interpret this as __** (1-digit number) **groups of __** (2-digit number). **It will be easier to set up the problem. So, we have __ groups of __. What do we have?**	
Students	__ groups of __.	
Teacher	**Let's use the Base-10 blocks to make __ groups of __. I'll make one group at a time.**	
	(Use Base-10 blocks to show groups with an equal number in each group.)	
Teacher	**Now, let's combine all the groups to learn the product. Let's put together all the ones.**	
	(Put together ones.)	
Teacher	**If we have more than 9 ones we have to regroup. Do we have more than 9 ones?**	
Students	*OPTION 1:*	No. We don't have to regroup.
	OPTION 2:	Yes. We have to regroup.
Teacher	***OPTION 2:***	**How do we group?**
Students		Regroup/trade/exchange 10 ones for 1 ten.
Teacher		**Let's exchange 10 ones for 1 ten. We'll leave the remaining ones and place the 1 ten with the tens.**
		(Regroup.)

Teacher	**Now, let's combine the tens.**	
	(Put together tens.)	
Teacher	**If we have more than 9 tens we have to regroup. Do we have more than 9 tens?**	
Students	*OPTION 1:*	No. We don't have to regroup.
	OPTION 2:	Yes. We have to regroup.
Teacher	***OPTION 2:***	**How do we group?**
Students		Regroup/trade/exchange 10 tens for 1 hundred.
Teacher		**Let's exchange 10 tens for 1 hundred. We'll leave the remaining tens and place the 1 hundred with the hundreds.**
		(Regroup.)
Teacher	**Now, let's combine the hundreds.**	
	(Put together hundreds.)	
Teacher	**If we have more than 9 hundreds we have to regroup. Do we have more than 9 hundreds?**	
Students	*OPTION 1:*	No. We don't have to regroup.
	OPTION 2:	Yes. We have to regroup.
Teacher	***OPTION 2:***	**How do we group?**
Students		Regroup/trade/exchange 10 hundreds for 1 thousand.
Teacher		**Let's exchange 10 hundreds for 1 thousand. We'll leave the remaining hundreds and place the 1 thousand with the thousands.**
		(Regroup.)
Teacher	**Let's count to determine the product.**	
	(Count the thousands, hundreds, tens, and ones.)	
Teacher	**That means __ times __ equals __. Let's say that together.**	
Students	__ times __ equals __.	
Teacher	**Let's say it together again.**	
Students	__ times __ equals __.	
Teacher	**So, if you have __ groups of __ and multiply by __, the product is __. __ times __ equals __. Let's review. What's a factor?**	
Students	The numbers multiplied in a multiplication problem.	
Teacher	**What's a product?**	
Students	The result of multiplying factors.	
Teacher	**What does it mean to make equal groups?**	
Students	To have groups with an equal number in each group.	
Teacher	**How could you explain multiplying to a friend?**	
Students	We used Base-10 blocks to make groups with the same number in each group. The product was the total number of blocks.	

2-DIGIT × 2-DIGIT: ROUTINE WITHOUT MANIPULATIVES

Teacher	**Let's work on multiplication. What does it mean to multiply?**

Students	To make equal groups or to compare.
Teacher	**Multiplication means to make equal groups or to compare. Look at this problem.** (Show problem.)
Teacher	**First, I see a multiplication sign** (point). **The multiplication sign tells us to multiply. What does the multiplication sign mean?**
Students	To multiply.
Teacher	**Let's do this problem with our pencil. First, when I see a problem like this that requires computation, I like to draw vertical lines to separate the ones from the tens. Let's draw a vertical line between the ones column and the tens column.** (Draw vertical lines to separate place value columns.)
Teacher	**Now, we start by multiplying the ones of the second factor. This means we'll write these products starting in the ones column below the equal line. Where will we write the products?**
Students	Below the equal line.
Teacher	**We first multiply the ones of the second factor times the ones of the first factor. What should we multiply first?**
Students	The ones of the second factor times the ones of the first factor.
Teacher	**Which ones do we multiply?**
Students	__ times __.
Teacher	**What's __ times __?** (If a student has difficulty with multiplication, use a multiplication table or other resource.)
Students	__.
Teacher	**__ times __ equals __. Let's write __ below the equal line in the ones column.** IF REGROUPING: **Our product is greater than 9, so we have to regroup. That means we write the ones in the ones place and regroup the tens.** (Write product.)
Teacher	**Now, we multiply the ones of the second factor times the tens of the first factor. What do we multiply?**
Students	The ones of the second factor times the tens of the first factor.
Teacher	**So, what do we multiply?**
Students	__ times __.
Teacher	**What's __ times __?**
Students	__.
Teacher	IF REGROUPING: **Remember, we regrouped __ from when we multiplied the ones of the second factor by the ones of the first factor. Now, we add that regrouped amount to our product of __ times __. So, what's __ plus __?** __.
Teacher	**Let's write __ below the equal line in the tens column.** (Write product.)

Teacher	**So, we multiplied the ones of the second factor times the ones of the first factor then the ones of the second factor times the tens of the first factor. Who can describe what we multiplied so far?**
Students	We multiplied the ones of the second factor times the ones of the first factor then times the tens of the first factor.
Teacher	**We've multiplied the ones of the second factor. Now, it's time to multiply the tens of the second factor. What will we multiply now?**
Students	The tens of the second factor.
Teacher	**When writing the products of multiplying the tens of the second factor, we'll write them below this first line of products. Because we're now multiplying by ten, we will write our products starting in the tens column. I like to place an X or zero in the ones column below the equal line to remember to start writing my products in the tens column.** (Write X or 0.)
Teacher	**Now, let's multiply the tens of the second factor times the ones of the first factor. What should we multiply?**
Students	The tens of the second factor times the ones of the first factor.
Teacher	**What numbers do we multiply?**
Students	__ times __.
Teacher	**What's __ times __?** (If a student has difficulty with multiplication, use a multiplication table or other resource.)
Students	__.
Teacher	**__ times __ equals __. Let's write __ below the equal line in the tens column.** IF REGROUPING: **Our product is greater than 9, so we have to regroup. That means we write the ones and regroup the tens above the problem.** (Write product.)
Teacher	**Now, we multiply the tens of the second factor times the tens of the first factor. What do we multiply?**
Students	The tens of the second factor times the tens of the first factor.
Teacher	**So, what do we multiply?**
Students	__ times __.
Teacher	**What's __ times __?**
Students	__.
Teacher	IF REGROUPING: **Remember, we regrouped __ from when we multiplied the tens of the second factor by the ones of the first factor. Now, we add that regrouped amount to our product of __ times __. So, what's __ plus __?**
Students	__.
Teacher	**Let's write __ below the equal line.** (Write product.)

Teacher	**So, we multiplied the tens of the second factor times the ones of the first factor and then the tens of the second factor times the tens of the first factor. Who can describe what we multiplied?**
Students	We multiplied the tens of the second factor times the ones of the first factor then times the tens of the first factor.
Teacher	**Now, we did all the multiplication but we are not finished! We call these numbers here** (point to numbers under equal line) **our partial products. We have to add the partial products together to determine the final product. Let's draw another equal line and write in a plus sign. What should we draw?**
Students	An equal line and plus sign. (Write equal line and plus sign.)
Teacher	**So, let's add __ plus __. What's __ plus __?** (If students need help with addition of whole numbers, see Module 5.)
Students	__.
Teacher	**Yes. So, I write __ under the equal line.** (Write final product.)
Teacher	**That means __ times __ equals __. Let's say that together.**
Students	__ times __ equals __.
Teacher	**Let's say it together again.**
Students	__ times __ equals __.
Teacher	**So, if you have __ and multiply by __, the product is __. __ times __ equals __. Let's review. What's a factor?**
Students	One of the numbers multiplied in a multiplication problem.
Teacher	**What's a product?**
Students	The result of multiplying factors.
Teacher	**What does it mean to make equal groups?**
Students	To have groups with an equal number in each group.
Teacher	**How could you explain multiplication of double-digit numbers to a friend?**
Students	We multiplied the ones of the second factor times the ones and tens of the first factor. Then, we multiplied the tens of the second factor times the ones and tens of the first factor. Finally, we added the partial products to determine the final product.

Example

$$\begin{array}{r} 13 \\ \times\ 45 \\ \hline 585 \end{array}$$

2-DIGIT × 2-DIGIT: EXAMPLE WITHOUT MANIPULATIVES

Teacher	**Let's work on multiplication. What does it mean to multiply?**
Students	To make equal groups or to compare.

Teacher	**Multiplication means to make equal groups or to compare. Look at this problem.** (Show problem.)
Teacher	**First, I see a multiplication sign** (point). **The multiplication sign tells us to multiply. What does the multiplication sign mean?**
Students	To multiply.
Teacher	**Let's do this problem with our pencil. First, when I see a problem like this that requires computation, I like to draw vertical lines to separate the ones from the tens. Let's draw a vertical line between the ones column and the tens column.** (Draw vertical lines to separate place value columns.)
Teacher	**Now, we start by multiplying the ones of the second factor. This means we'll write these products starting in the ones column below the equal line. Where will we write the products?**
Students	Below the equal line.
Teacher	**We first multiply the ones of the second factor times the ones of the first factor. What should we multiply first?**
Students	The ones of the second factor times the ones of the first factor.
Teacher	**What are the ones of the second factor?**
Students	5.
Teacher	**What are the ones of the first factor?**
Students	3.
Teacher	**So, we'll multiply 5 times 3. What do we multiply?**
Students	5 times 3.
Teacher	**What's 5 times 3?** (If a student has difficulty with multiplication, use a multiplication table or other resource.)
Students	15.
Teacher	**5 times 3 equals 15. 15 is greater than 9, so we have to regroup. That means we write the 5 of 15 in the ones place below the equal line. We write the 1 of 15 above the tens column.** (Write product.)
Teacher	**Now, we multiply the ones of the second factor times the tens of the first factor. What do we multiply?**
Students	The ones of the second factor times the tens of the first factor.
Teacher	**So, what do we multiply?**
Students	5 times 1.
Teacher	**What's 5 times 1?**
Students	5.
Teacher	**Is the product greater than 9?**
Students	No.
Teacher	**Do we have to regroup?**
Students	No.

Teacher	**But we do have to remember to add the regrouped amount to our product. That means we'll add 5 plus 1. What's 5 plus 1?**
Students	6.
Teacher	**Let's write 6 below the equal line in the tens column.** (Write product.)
Teacher	**So, we multiplied the ones of the second factor times the ones then the tens. Who can describe what we multiplied so far?**
Students	We multiplied 5 times 3. Then we multiplied 5 times 1.
Teacher	**We've multiplied the ones of the second factor. Now, it's time to multiply the tens of the second factor. What will we multiply now?**
Students	The tens of the second factor.
Teacher	**When writing the products of multiplying the tens of the second factor, we'll write them below this first line of products. Because we're now multiplying by ten, we will write our products starting in the tens column. I like to place an X or zero in the ones column below the equal line to remember to start writing my products in the tens column.** (Write X or 0.)
Teacher	**Now, let's multiply the tens of the second factor times the ones of the first factor. What should we multiply?**
Students	The tens of the second factor times the ones of the first factor.
Teacher	**What are the tens of the second factor?**
Students	4.
Teacher	**What are the ones of the first factor?**
Students	3.
Teacher	**So, we'll multiply 4 times 3. What do we multiply?**
Students	4 times 3.
Teacher	**What's 4 times 3?** (If a student has difficulty with multiplication, use a multiplication table or other resource.)
Students	12.
Teacher	**4 times 3 equals 12. 12 is greater than 9, so we have to regroup. That means we write the 2 of 12 in the tens place below the equal line. We write the 1 of 12 above the hundreds column.** (Write product.)
Teacher	**Now, we multiply the tens of the second factor times the tens of the first factor. What do we multiply?**
Students	The tens of the second factor times the tens of the first factor.
Teacher	**So, what do we multiply?**
Students	4 times 1.
Teacher	**What's 4 times 1?**
Students	4.
Teacher	**Is the product greater than 9?**
Students	No.
Teacher	**Do we have to regroup?**

Students	No.
Teacher	**But we do have to remember to add the regrouped amount to our product. That means we'll add 4 plus 1. What's 4 plus 1?**
Students	5.
Teacher	**Let's write 5 below the equal line in the tens column.** (Write product.)
Students	__.
Teacher	**Let's write __ below the equal line.** (Write product.)
Teacher	**So, we multiplied the tens of the second factor times the ones of the first factor then the tens of the first factor. Who can describe what we multiplied?**
Students	We multiplied 4 times 3 then 4 times 1.
Teacher	**We did the multiplication. Are we finished?**
Students	No!
Teacher	**We are not finished! We call these numbers here** (point to numbers under equal line) **our partial products. We have to add the partial products together to determine the final product. Let's draw another equal line and write in a plus sign. What should we draw?**
Students	An equal line and plus sign. (Write equal line and plus sign.)
Teacher	**So, let's add 65 plus 520. What's 65 plus 520?** (If students need help with addition of whole numbers, see Module 5.)
Students	585.
Teacher	**Yes. So, I write 585 under the equal line.** (Write final product.)
Teacher	**That means 13 times 45 equals 585. Let's say that together.**
Students	13 times 45 equals 585.
Teacher	**Let's say it together again.**
Students	13 times 45 equals 585.
Teacher	**So, if you have 13 and multiply by 45, the product is 585. Let's review. What's a factor?**
Students	One of the numbers multiplied in a multiplication problem.
Teacher	**What's a product?**
Students	The result of multiplying factors.
Teacher	**What does it mean to make equal groups?**
Students	To have groups with an equal number in each group.
Teacher	**How could you explain multiplication of double-digit numbers to a friend?**
Students	We multiplied the ones of the second factor first. That meant we multiplied 5 times 3 then 5 times 1. Then, we multiplied the tens of the second factor. We multiplied 4 times 3 then 4 times 1. Finally, we added the partial products of 65 and 520 to determine the product of 585.

(2) Multiplication with Partial Products Algorithm*

*For clarity, read **Example** before using **Routines**.

Routine

Materials:

- Module 11 Problem Sets
- Module 11 Vocabulary Cards
    - If necessary, review Vocabulary Cards before teaching
- A hands-on tool or manipulative like Base-10 blocks or unifix cubes
    - Note that drawings can be used alongside or instead of manipulatives

2-DIGIT × 1-DIGIT: ROUTINE WITH MANIPULATIVES

(Only use manipulatives with simpler problems)

Teacher	**Let's work on multiplication. What does it mean to multiply?**
Students	To make equal groups or to compare.
Teacher	**Multiplication means to make equal groups or to compare. Look at this problem.** (Show problem.)
Teacher	**First, I see a multiplication sign** (point). **The multiplication sign tells us to multiply. What does the multiplication sign mean?**
Students	To multiply.
Teacher	**Let's do this problem with Base-10 blocks.** (Move Base-10 blocks to workspace.)
Teacher	**With our Base-10 blocks, the flats represent hundreds. What do the flats represent?**
Students	Hundreds.
Teacher	**The rods represent tens. What do the rods represent?**
Students	Tens.
Teacher	**With our Base-10 blocks, the units represent ones. What do the units represent?**
Students	Ones.
Teacher	**Our first factor is __. What's our first factor?**
Students	__.
Teacher	**Our second factor is __. What's our second factor?**
Students	__.
Teacher	**Let's solve this problem using multiplication as equal groups. What does equal groups mean?**
Students	We have groups with an equal number in each group.
Teacher	**We will use the partial products strategy to solve this problem. Say partial products.**
Students	Partial products.

Teacher	**With the partial products strategy, we do the multiplication for each factor then we add the partial products together for a final product. With the partial products strategy, we work from the greatest place value to the least place value. How do we work?**
Students	From the greatest place value to the least place value.
Teacher	**In this problem, what is the greatest place value?**
Students	Tens.
Teacher	**The tens are the greatest place value, so we'll start by multiplying the ones of the second factor by the tens of the first factor. Where do we start?**
Students	By multiplying the ones of the second factor times the tens of the first factor.
Teacher	**First, let's multiply the ones of the second factor times the tens of the first factor. What are the tens of the first factor?**
Students	__.
Teacher	**We have __ tens. __ tens is the same as what?**
Students	__.
Teacher	**So, we multiply __ times __. Let's use the Base-10 blocks to make __ groups of __. I'll make one group at a time.** (Use Base-10 blocks to show groups with an equal number in each group.)
Teacher	**These Base-10 blocks are one of our partial products. Now, let's multiply the ones of the second factor times the ones of the first factor. What are the ones of the second factor?**
Students	__.
Teacher	**Let's then multiply __ times __. Let's use the Base-10 blocks to make __ groups of __. I'll make one group at a time.** (Use Base-10 blocks to show groups with an equal number in each group.)
Teacher	**This group of Base-10 blocks is another partial product. Now, let's add all the partial products, or Base-10 blocks, to determine the final product.** (Count the hundreds, tens, and ones.)
Teacher	**That means __ times __ equals __. Let's say that together.**
Students	__ times __ equals __.
Teacher	**Let's say it together again.**
Students	__ times __ equals __.
Teacher	**So, if you have __ groups of __ and multiply by __, the product is __. __ times __ equals __. Let's review. What's a factor?**
Students	The numbers multiplied in a multiplication problem.
Teacher	**What's a product?**
Students	The result of multiplying factors.
Teacher	**What does it mean to use the partial products strategy?**
Students	We multiplied each factor for a partial product. Then, we added the partial products to determine the final product.
Teacher	**How could you explain multiplying to a friend?**
Students	We multiplied the ones of the second factor times the tens of the first factor. Then, we multiplied the ones of the second factor times the ones of the first factor. We added the partial products to determine the final product.

2-DIGIT × 2-DIGIT: ROUTINE WITHOUT MANIPULATIVES

Teacher **Let's work on multiplication. What does it mean to multiply?**

Students To make equal groups or to compare.

Teacher **Multiplication means to make equal groups or to compare. Look at this problem.**

(Show problem.)

Teacher **First, I see a multiplication sign** (point). **The multiplication sign tells us to multiply. What does the multiplication sign mean?**

Students To multiply.

Teacher **Let's do this problem with our pencil. First, when I see a problem like this that requires computation, I like to draw vertical lines to separate the ones from the tens. Let's draw a vertical line between the ones column and the tens column.**

(Draw vertical lines to separate place value columns.)

Teacher **Let's use the partial products strategy. What strategy?**

Students Partial products.

Teacher **With the partial products strategy, we do the multiplication for each factor then we add the partial products together for a final product. With the partial products strategy, we work from the greatest place value to the least place value. How do we work?**

Students From the greatest place value to the least place value.

Teacher **First, we'll multiply the tens of the second factor times the tens of the first factor and ones of the first factor. Let's do that now. What are the tens of the second factor?**

Students __.

Teacher **We have __ tens in the second factor. __ tens is the same as what?**

Students __.

Teacher **Look at the first factor. What are the tens of the first factor?**

Students __.

Teacher **We have __ tens in the first factor. __ tens is the same as what?**

Students __.

So, let's multiply __ times __. What's __ times __?

Students __.

Teacher **__ times __ equals __. Let's write __ below the equal line.**

(Write product.)

Teacher **__ is our first partial product. Now, let's multiply the tens of the second factor times the ones of the first factor? What do we multiply?**

Students __ times __.

Teacher **What's __ times __?**

Students __.

Teacher	**Let's write __ below the equal line. We'll write this second partial product under the first partial product.**
	(Write product.)
Teacher	**Now, let's multiply the ones of the second factor times the tens of the first factor and ones of the first factor. Let's do that now. What are the ones of the second factor?**
Students	__.
Teacher	**We have __ ones in the second factor. Look at the first factor. What are the tens of the first factor?**
Students	__.
Teacher	**We have __ tens in the first factor. __ tens is the same as what?**
Students	__.
	So, let's multiply __ times __. What's __ times __?
Students	__.
Teacher	**__ times __ equals __. Let's write __ below the equal line under our other partial products.**
	(Write product.)
Teacher	**Finally, let's multiply the ones of the second factor times the ones of the first factor. What do we multiply?**
Students	__ times __.
Teacher	**What's __ times __?**
Students	__.
Teacher	**Let's write __ below the equal line under our other partial products.**
	(Write product.)
Teacher	**To determine the final product, we add all the partial products together. I'll write a plus sign and another equal line.**
	(Write plus sign and equal line.)
Teacher	**So, what's __ plus __ plus __ plus __?**
	(For assistance with the partial sums algorithm for addition, see Module 5.)
Students	__.
Teacher	**__ is our final product. Let's write __ under the equal line.**
Students	(Write product.)
Teacher	**That means __ times __ equals __. Let's say that together.**
Students	__ times __ equals __.
Teacher	**Let's say it together again.**
Students	__ times __ equals __.
Teacher	**So, if you have __ groups and multiply by __, the product is __. __ times __ equals __. Let's review. What's a factor?**
Students	The numbers multiplied in a multiplication problem.
Teacher	**What's a product?**
Students	The result of multiplying factors.
Teacher	**What does it mean to use the partial products strategy?**
Students	We multiplied each factor for a partial product. That means we multiplied the tens of the second factor times the tens of the first factor then the ones of the

first factor. We also multiplied the ones of the second factor times the tens of the first factor and ones of the first factor. Then, we added the partial products to determine the final product.

Teacher **How could you explain multiplying to a friend?**

Students We multiplied the tens of the second factor times the tens and ones of the first factor. Then, we multiplied the ones of the second factor times the tens and ones of the first factor. We added the partial products to determine the final product.

Example

$$\begin{array}{r} 13 \\ \times\ 45 \\ \hline 400 \\ 120 \\ 50 \\ +\ 15 \\ \hline 585 \end{array}$$

2-DIGIT × 2-DIGIT: EXAMPLE WITHOUT MANIPULATIVES

Teacher **Let's work on multiplication. What does it mean to multiply?**

Students To make equal groups or to compare.

Teacher **Multiplication means to make equal groups or to compare. Look at this problem.**

(Show problem.)

Teacher **First, I see a multiplication sign** (point). **The multiplication sign tells us to multiply. What does the multiplication sign mean?**

Students To multiply.

Teacher **Let's do this problem with our pencil. First, when I see a problem like this that requires computation, I like to draw vertical lines to separate the ones from the tens. Let's draw a vertical line between the ones column and the tens column.**

(Draw vertical lines to separate place value columns.)

Teacher **Let's use the partial products strategy. What strategy?**

Students Partial products.

Teacher **With the partial products strategy, we do the multiplication for each factor then we add the partial products together for a final product. With the partial products strategy, we work from the greatest place value to the least place value. How do we work?**

Students From the greatest place value to the least place value.

Teacher	**First, we'll multiply the tens of the second factor times the tens of the first factor and ones of the first factor. Let's do that now. What are the tens of the second factor?**
Students	4.
Teacher	**We have 4 tens in the second factor. 4 tens is the same as what?**
Students	40.
Teacher	**Look at the first factor. What are the tens of the first factor?**
Students	1.
Teacher	**We have 1 ten in the first factor. 1 ten is the same as what?**
Students	10.
	So, let's multiply 40 times 10. What's 40 times 10?
Students	400.
Teacher	**40 times 10 equals 400. Let's write 400 below the equal line.** (Write product.)
Teacher	**400 is our first partial product. Now, let's multiply the tens of the second factor times the ones of the first factor? What do we multiply?**
Students	40 times 3.
Teacher	**What's 40 times 3?**
Students	120.
Teacher	**Let's write 120 below the equal line. We'll write this partial product under the first partial product.** (Write product.)
Teacher	**Now, let's multiply the ones of the second factor times the tens of the first factor and ones of the first factor. Let's do that now. What are the ones of the second factor?**
Students	5.
Teacher	**We have 5 ones in the second factor. Look at the first factor. What are the tens of the first factor?**
Students	1.
Teacher	**We have 1 ten in the first factor. 1 ten is the same as what?**
Students	10.
	So, let's multiply 5 times 10. What's 5 times 10?
Students	50.
Teacher	**5 times 10 equals 50. Let's write 50 below the equal line under our other partial products.** (Write product.)
Teacher	**Finally, let's multiply the ones of the second factor times the ones of the first factor. What do we multiply?**
Students	5 times 3.
Teacher	**What's 5 times 3?**
Students	15.
Teacher	**Let's write 15 below the equal line under our other partial products.** (Write product.)

Teacher	**To determine the final product, we add all the partial products together. I'll write a plus sign and another equal line.**
	(Write plus sign and equal line.)
Teacher	**I like to add in steps. What's 400 plus 120?**
Students	520.
Teacher	**What's 520 plus 50?**
Students	570.
Teacher	**What's 570 plus 15?**
Students	585.
Teacher	**585 is our final product. Let's write 585 under the equal line.**
Students	(Write product.)
Teacher	**That means 13 times 45 equals 585. Let's say that together.**
Students	13 times 45 equals 585.
Teacher	**Let's say it together again.**
Students	13 times 45 equals 585.
Teacher	**So, if you have 13 and multiply by 45, the product is 585. 13 times 45 equals 585. Let's review. What's a factor?**
Students	The numbers multiplied in a multiplication problem.
Teacher	**What's a product?**
Students	The result of multiplying factors.
Teacher	**What does it mean to use the partial products strategy?**
Students	We multiplied each factor for a partial product. Then, we added the partial products to determine the final product.
Teacher	**How could you explain multiplying to a friend?**
Students	We multiply 40 times 10, then 40 times 3. Then, we multiplied 5 times 10 and 5 times 3. We added the partial products for a final product of 585.

(3) Multiplication with Array (Area Model)

*For clarity, read **Example** before using **Routine**.

Routine

Materials:

- Module 11 Problem Sets
- Module 11 Vocabulary Cards
    - If necessary, review Vocabulary Cards before teaching

2-DIGIT × 2-DIGIT: ROUTINE WITHOUT MANIPULATIVES

Teacher	**Let's work on multiplication. What does it mean to multiply?**
Students	To make equal groups or to compare.
Teacher	**Multiplication means to make equal groups or to compare. Look at this problem.** (Show problem.)
Teacher	**First, I see a multiplication sign** (point). **The multiplication sign tells us to multiply. What does the multiplication sign mean?**
Students	To multiply.
Teacher	**Let's do this problem using the array model. We'll create an array or rectangular area with our multiplication problem. The array model is similar to the partial products model. Let's get started. First, I have to draw rectangular array. What do I have to draw?**
Students	Rectangular array.
Teacher	**My array includes the place value of each factor. How many digits in the first factor?**
Students	__.
Teacher	**So, that's a __-digit factor. How many digits in the second factor?**
Students	__.
Teacher	**So, that's a __-digit factor. Our array should have __ columns for the first factor and __ rows for the second factor. Let's draw an array with __ columns and __ rows.** (Draw array.)
Teacher	**Now, I write the first factor in expanded form. What does expanded form mean?**
Students	To write the number in tens and ones.
Teacher	**How many tens are in the first factor?**
Students	__.
Teacher	**__ tens is the same as __. So the expanded form of __ would be __ plus __. Let's write __ and __ above the columns.** (Write first factor in expanded form.)
Teacher	**Now, I write the second factor in expanded form on the right side of the array. What does expanded form mean?**

Students	To write the number in tens and ones.
Teacher	**How many tens are in the second factor?**
Students	__.
Teacher	**__ tens is the same as __. So the expanded form of __ would be __ plus __. Let's write __ and __ next to the row on the right side.** (Write second factor in expanded form.)
Teacher	**Now that we have set up the problem, let's multiply. I like to multiply the second factor times the first factor but any order is okay – the commutative property helps us with that! Let's multiply __** (tens on row) **times __** (tens on column.) **What's __ times __?**
Students	__.
Teacher	**__ times __ equals __. Let's write __ in the part of the array in which the row and column meet.** (Write product.)
Teacher	**__ is a partial product. Now, let's multiply the tens of the second factor times the ones of the first factor. What do we multiply?**
Students	__ times __.
Teacher	**What's __ times __?**
Students	__.
Teacher	**Let's write __ in the part of the array in which the row and column meet.** (Write product.)
Teacher	**Now, let's multiply the ones of the second factor times the tens of the first factor and ones of the first factor. Let's do that now. What are the ones of the second factor?**
Students	__.
Teacher	**We have __ ones in the second factor. Look at the first factor. What are the tens of the first factor?**
Students	__.
Teacher	**We have __ tens in the first factor. __ tens is the same as what?**
Students	__. **So, let's multiply __ times __. What's __ times __?**
Students	__.
Teacher	**__ times __ equals __. Let's write __ in the part of the array in which the row and column meet.** (Write product.)
Teacher	**Finally, let's multiply the ones of the second factor times the ones of the first factor. What do we multiply?**
Students	__ times __.
Teacher	**What's __ times __?**
Students	__.
Teacher	**Let's write __ in the part of the array in which the row and column meet.** (Write product.)
Teacher	**To determine the final product, we add all the partial products together. I'll write all the partial products from greatest to least.**

(Rewrite partial products.)

Teacher **So, what's __ plus __ plus __ plus __?**

(For assistance with the partial sums algorithm for addition, see Module 5.)

Students __.

Teacher **__ is our final product. Let's write __ under the equal line.**

Students (Write product.)

Teacher **That means __ times __ equals __. Let's say that together.**

Students __ times __ equals __.

Teacher **Let's say it together again.**

Students __ times __ equals __.

Teacher **So, if you have __ groups and multiply by __, the product is __. __ times __ equals __. Let's review. What's a factor?**

Students The numbers multiplied in a multiplication problem.

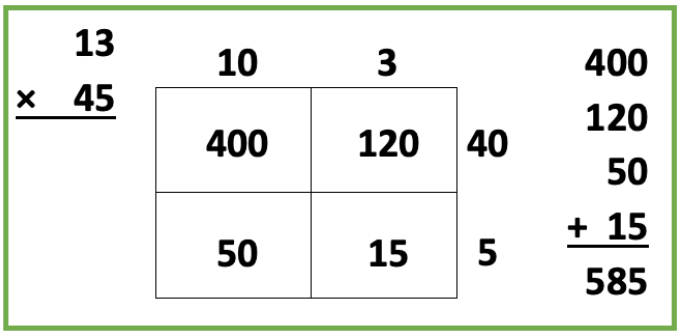
Teacher **What's a product?**

Students The result of multiplying factors.

Teacher **What does it mean to use an array?**

Students We determined the expanded form for each factor. Then, we multiplied each factor for a partial product. Finally, we added the partial products to determine the final product.

Example



2-DIGIT × 2-DIGIT: EXAMPLE WITHOUT MANIPULATIVES

Teacher **Let's work on multiplication. What does it mean to multiply?**

Students To make equal groups or to compare.

Teacher **Multiplication means to make equal groups or to compare. Look at this problem.**

(Show problem.)

Teacher **First, I see a multiplication sign** (point). **The multiplication sign tells us to multiply. What does the multiplication sign mean?**

Students To multiply.

Teacher **Let's do this problem using the array model. We'll create an array or rectangular area with our multiplication problem. The array model is similar to the partial products model. Let's get started. First, I have to draw rectangular array. What do I have to draw?**

Students	Rectangular array.
Teacher	**My array includes the place value of each factor. How many digits in the first factor?**
Students	2.
Teacher	**So, that's a 2-digit factor. How many digits in the second factor?**
Students	2.
Teacher	**So, that's a 2-digit factor. Our array should have 2 columns for the first factor and 2 rows for the second factor. Let's draw an array with 2 columns and 2 rows.**
	(Draw array.)
Teacher	**Now, I write the first factor in expanded form. What does expanded form mean?**
Students	To write the number in tens and ones.
Teacher	**How many tens are in the first factor?**
Students	1.
Teacher	**1 ten is the same as 10. So, the expanded form of 13 would be 10 plus 3. Let's write 10 and 3 above the columns.**
	(Write first factor in expanded form.)
Teacher	**Now, I write the second factor in expanded form on the right side of the array. What does expanded form mean?**
Students	To write the number in tens and ones.
Teacher	**How many tens are in the second factor?**
Students	4.
Teacher	**4 tens is the same as 40. So, the expanded form of 45 would be 40 plus 5. Let's write 40 and 5 next to the row on the right side.**
	(Write second factor in expanded form.)
Teacher	**Now that we have set up the problem, let's multiply. I like to multiply the second factor times the first factor but any order is okay – the commutative property helps us with this! Let's multiply 40 times 10. What's 40 times 10?**
Students	400.
Teacher	**40 times 10 equals 400. Let's write 400 in the part of the array in which the row and column meet.**
	(Write product.)
Teacher	**400 is a partial product. Now, let's multiply the tens of the second factor times the ones of the first factor. What do we multiply?**
Students	40 times 3.
Teacher	**What's 40 times 3?**
Students	120.
Teacher	**Let's write 120 in the part of the array in which the row and column meet.**
	(Write product.)
Teacher	**Now, let's multiply the ones of the second factor times the tens of the first factor and ones of the first factor. Let's do that now. What are the ones of the second factor?**
Students	5.

Teacher	**We have 5 ones in the second factor. Look at the first factor. What are the tens of the first factor?**
Students	1.
Teacher	**We have 1 ten in the first factor. 1 ten is the same as what?**
Students	10.
	So, let's multiply 5 times 10. What's 5 times 10?
Students	50.
Teacher	**5 times 10 equals 50. Let's write 50 in the part of the array in which the row and column meet.**
	(Write product.)
Teacher	**Finally, let's multiply the ones of the second factor times the ones of the first factor. What do we multiply?**
Students	5 times 3.
Teacher	**What's 5 times 3?**
Students	15.
Teacher	**Let's write 15 in the part of the array in which the row and column meet.**
	(Write product.)
Teacher	**To determine the final product, we add all the partial products together. I'll write all the partial products from greatest to least.**
	(Rewrite to 400 + 120 + 50 + 15.)
Teacher	**Let's add this in steps. What's 400 plus 120?**
Students	520.
Teacher	**What's 520 plus 50?**
Students	570.
Teacher	**What's 570 plus 15?**
Students	585.
Teacher	**585 is our final product. Let's write 585 under the equal line.**
Students	(Write product.)
Teacher	**That means 13 times 45 equals 585. Let's say that together.**
Students	13 times 45 equals 585.
Teacher	**Let's say it together again.**
Students	13 times 45 equals 585.
Teacher	**So, if you have 13 and multiply by 45, the product is 585. 13 times 45 equals 585. Let's review. What's a factor?**
Students	The numbers multiplied in a multiplication problem.
Teacher	**What's a product?**
Students	The result of multiplying factors.
Teacher	**What does it mean to use an array?**
Students	We determined the expanded form for each factor. Then, we multiplied each factor for a partial product. Finally, we added the partial products to determine the final product.

D. Problems for Use During Instruction

See Module 11 Problem Sets.

E. Vocabulary Cards for Use During Instruction

See Module 11 Vocabulary Cards.

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Module 11:
Multiplication of Whole Numbers

Problem Sets

A. Two-digit numbers by one-digit numbers (30)
B. Two-digit numbers by two-digit numbers (30)
C. Three-digit numbers by two-digit numbers (20)

A.

$$\begin{array}{r} 49 \\ \times \quad 3 \\ \hline \end{array}$$

A.

$$\begin{array}{r} 76 \\ \times \quad 4 \\ \hline \end{array}$$

A.

$$\begin{array}{r} 25 \\ \times \quad 2 \\ \hline \\ \end{array}$$

A.

$$\begin{array}{r} 33 \\ \times \quad 8 \\ \hline \end{array}$$

A.

$$\begin{array}{r} 18 \\ \times \quad 7 \\ \hline \end{array}$$

A.

$$\begin{array}{r} 54 \\ \times \quad 8 \\ \hline \end{array}$$

A.

$$\begin{array}{r} 69 \\ \times \quad 3 \\ \hline \end{array}$$

A.

$$\begin{array}{r} 42 \\ \times \quad 5 \\ \hline \end{array}$$

A.

$$\begin{array}{r} 84 \\ \times \quad 2 \\ \hline \end{array}$$

A.

$$\begin{array}{r} 75 \\ \times \quad 7 \\ \hline \end{array}$$

A.

$$\begin{array}{r} 56 \\ \times \quad 3 \\ \hline \end{array}$$

A.

$$\begin{array}{r} 92 \\ \times \quad 6 \\ \hline \end{array}$$

A.

$$\begin{array}{r} 41 \\ \times \quad 9 \\ \hline \end{array}$$

A.

$$\begin{array}{r} 72 \\ \times \quad 8 \\ \hline \end{array}$$

A.

$$\begin{array}{r} 67 \\ \times \quad 9 \\ \hline \end{array}$$

A.

$$\begin{array}{r} 33 \\ \times \quad 7 \\ \hline \end{array}$$

A.

$$\begin{array}{r} 48 \\ \times \quad 5 \\ \hline \end{array}$$

A.

$$\begin{array}{r} 56 \\ \times \quad 6 \\ \hline \end{array}$$

A.

$$\begin{array}{r} 30 \\ \times \quad 2 \\ \hline \end{array}$$

A.

$$\begin{array}{r} 86 \\ \times \quad 5 \\ \hline \end{array}$$

A.

$$\begin{array}{r} 34 \\ \times \quad 7 \\ \hline \end{array}$$

A.

$$\begin{array}{r} 60 \\ \times \quad 5 \\ \hline \end{array}$$

A.

$$\begin{array}{r} 53 \\ \times \quad 4 \\ \hline \end{array}$$

A.

$$\begin{array}{r} 90 \\ \times \quad 8 \\ \hline \end{array}$$

A.

$$\begin{array}{r} 83 \\ \times \quad 4 \\ \hline \\ \\ \end{array}$$

A.

$$\begin{array}{r} 21 \\ \times \quad 9 \\ \hline \end{array}$$

A.

$$\begin{array}{r} 10 \\ \times \quad 3 \\ \hline \end{array}$$

A.

$$\begin{array}{r} 11 \\ \times\ 8 \\ \hline \end{array}$$

A.

$$\begin{array}{r} 27 \\ \times \quad 3 \\ \hline \end{array}$$

A.

$$\begin{array}{r} 42 \\ \times \quad 8 \\ \hline \end{array}$$

B.

$$\begin{array}{r} 44 \\ \times\ 46 \\ \hline \end{array}$$

B.

$$\begin{array}{r} 72 \\ \times\ 19 \\ \hline \end{array}$$

B.

$$\begin{array}{r} 48 \\ \times\ 49 \\ \hline \end{array}$$

B.

$$\begin{array}{r} 13 \\ \times\ 90 \\ \hline \end{array}$$

B.

$$\begin{array}{r} 46 \\ \times\ 16 \\ \hline \end{array}$$

B.

$$\begin{array}{r} 61 \\ \times\ 10 \\ \hline \end{array}$$

B.

$$\begin{array}{r} 25 \\ \times\ 55 \\ \hline \end{array}$$

B.

$$\begin{array}{r} 41 \\ \times\ 63 \\ \hline \end{array}$$

B.

$$\begin{array}{r} 97 \\ \times\ 42 \\ \hline \end{array}$$

B.

$$\begin{array}{r} 36 \\ \times\ 56 \\ \hline \end{array}$$

B.

$$\begin{array}{r} 48 \\ \times\ 15 \\ \hline \end{array}$$

B.

$$\begin{array}{r} 77 \\ \times 88 \\ \hline \end{array}$$

B.

$$\begin{array}{r} 84 \\ \times\ 84 \\ \hline \end{array}$$

B.

$$\begin{array}{r} 59 \\ \times\ 18 \\ \hline \end{array}$$

B.

$$\begin{array}{r} 28 \\ \times\ 25 \\ \hline \end{array}$$

B.

$$\begin{array}{r} 81 \\ \times\ 30 \\ \hline \end{array}$$

B.

$$\begin{array}{r} 14 \\ \times\ 57 \\ \hline \end{array}$$

B.

$$\begin{array}{r} 68 \\ \times\ 41 \\ \hline \end{array}$$

B.

$$\begin{array}{r} 34 \\ \times\ 45 \\ \hline \end{array}$$

B.

$$\begin{array}{r} 99 \\ \times\ 92 \\ \hline \end{array}$$

B.

$$\begin{array}{r} 78 \\ \times\ 43 \\ \hline \end{array}$$

B.

$$\begin{array}{r} 94 \\ \times\ 12 \\ \hline \end{array}$$

B.

$$\begin{array}{r} 14 \\ \times\ 33 \\ \hline \end{array}$$

B.

$$\begin{array}{r} 56 \\ \times\ 77 \\ \hline \end{array}$$

B.

$$\begin{array}{r} 84 \\ \times\ 24 \\ \hline \end{array}$$

B.

$$\begin{array}{r} 16 \\ \times\ 51 \\ \hline \end{array}$$

B.

$$\begin{array}{r} 65 \\ \times\ 12 \\ \hline \end{array}$$

B.

$$\begin{array}{r} 89 \\ \times\ 47 \\ \hline \end{array}$$

B.

$$\begin{array}{r} 22 \\ \times\ 88 \\ \hline \\ \end{array}$$

B.

$$\begin{array}{r} 76 \\ \times\ 20 \\ \hline \end{array}$$

c.

$$\begin{array}{r} 529 \\ \times\ 65 \\ \hline \end{array}$$

c.

$$\begin{array}{r} 273 \\ \times\ 86 \\ \hline \end{array}$$

c.

$$\begin{array}{r} 300 \\ \times \quad 73 \\ \hline \end{array}$$

c.

$$\begin{array}{r} 101 \\ \times \quad 67 \\ \hline \end{array}$$

c.

$$\begin{array}{r} 904 \\ \times \quad 51 \\ \hline \end{array}$$

c.

$$\begin{array}{r} 616 \\ \times \quad 41 \\ \hline \end{array}$$

c.

$$\begin{array}{r} 504 \\ \times \quad 88 \\ \hline \end{array}$$

c.

$$\begin{array}{r} 187 \\ \times \quad 59 \\ \hline \end{array}$$

c.

$$\begin{array}{r} 720 \\ \times \quad 89 \\ \hline \end{array}$$

c.

$$\begin{array}{r} 860 \\ \times \quad 22 \\ \hline \end{array}$$

c.

$$\begin{array}{r} 749 \\ \times \quad 15 \\ \hline \end{array}$$

c.

$$\begin{array}{r} 295 \\ \times \quad 35 \\ \hline \end{array}$$

c.

$$\begin{array}{r} 257 \\ \times \quad 21 \\ \hline \end{array}$$

c.

$$\begin{array}{r} 193 \\ \times \quad 57 \\ \hline \end{array}$$

c.

$$\begin{array}{r} 236 \\ \times\ 98 \\ \hline \end{array}$$

c.

$$\begin{array}{r} 399 \\ \times \quad 43 \\ \hline \end{array}$$

c.

$$\begin{array}{r} 244 \\ \times \quad 14 \\ \hline \end{array}$$

c.

$$\begin{array}{r} 660 \\ \times \quad 63 \\ \hline \end{array}$$

c.

$$\begin{array}{r} 879 \\ \times\ 62 \\ \hline \end{array}$$

c.

$$\begin{array}{r} 523 \\ \times \quad 12 \\ \hline \end{array}$$

Module 11:

Multiplication of Whole Numbers

Vocabulary Cards

algorithm
area
array
computation
commutative property
equal groups
equal sign
factor
hundreds column
multiply/multiplication
multiplication sign
ones column
partial products
product
regroup/trade/exchange
tens column

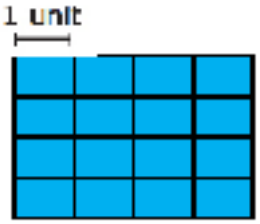
algorithm

A procedure or description of steps that can be used to solve a problem.

---

area

The number of square units that covers a closed figure.



array

A set of objects, pictures, ed in columns and rows.

---

computation

The action used to solve a problem.

commutative property (of multiplication)

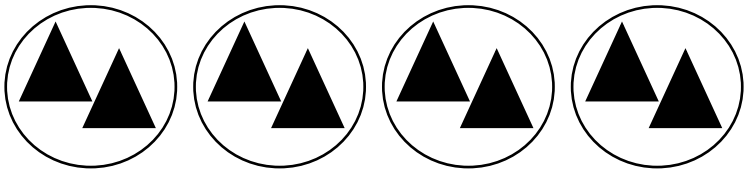
Two factors can be multiplied in any order.

$$2 \times 8 = 8 \times 2$$

---

equal groups

Groups with the same number of objects or items in each group.



equal sign

The symbol that tells you that two sides of an equation are the same, balanced, or equal.

$$2 \times 8 = 16$$

= is the equal sign

---

factor

A number you multiply with another number to get the product.

$$2 \times 8 = 16$$

2 and 8 are the factors

hundreds column

The column with digits in the hundreds place.

In the number 423, 4 is in the hundreds place.

---

multiply/multiplication

The process of adding a number to itself a number of times.

$$4 \times 2 = 8$$

multiplication sign

The symbol that tells you to multiply.

$$2 \times 8 = 16$$

× is the multiplication sign

---

ones column

The column with digits in the ones place.

In the number 423, 3 is in the ones place.

partial products

The product of parts of each factor.

$$
\begin{array}{r}
13 \\
\times\ 45 \\
\hline
400\ (40 \times 10) \\
120\ (40 \times 3) \\
50\ (10 \times 5) \\
+\ 15\ (5 \times 3) \\
\hline
585
\end{array}
$$

---

product

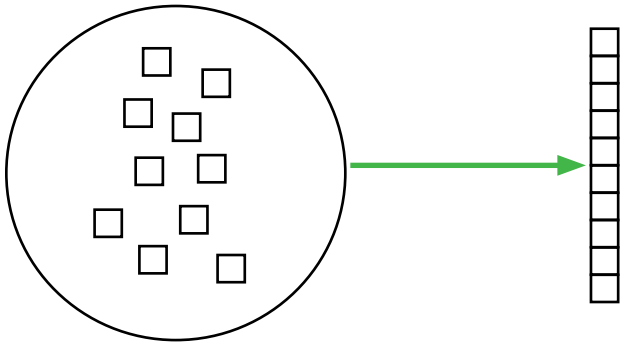
The result of multiplying two or more factors.

$$2 \times 8 = 16$$

16 is the product

regroup/trade/exchange

The process of exchanging 10 ones for 1 ten, 10 tens for 1 hundred, 10 hundreds for 1 thousand, etc.



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tens column

The column with digits in the tens place.

In the number 423, 2 is the in the tens column.