Instructional Routines for Mathematics Intervention

The purpose of these mathematics instructional routines is to provide educators with materials to use when providing intervention to students who experience difficulty with mathematics. The routines address content included in the grades 2-8 Texas Essential Knowledge and Skills (TEKS). There are 23 modules that include routines and examples – each focused on different mathematical content. Each of the 23 modules include vocabulary cards and problem sets to use during instruction. These materials are intended to be implemented explicitly with the aim of improving mathematics outcomes for students.

TEA
Texas Education Agency

Instructional Routines for Mathematics Intervention

MODULE 15

Division of Rational Numbers

Module 15: Division of Rational Numbers
Mathematics Routines

A. Important Vocabulary with Definitions

Term	Definition
algorithm	A procedure or description of steps that can be used to solve a problem.
computation	The action used to solve a problem.
decimal	A number based on powers of ten.
denominator	The term in a fraction that tells the number of equal parts in a whole.
divide/division	To separate into equal groups.
dividend	The number that is to be divided in a division problem.
division sign	The symbol that tells you to divide.
divisor	The number that the dividend is divided by.
equal groups	Groups with the same number of objects or items in each group.
equal sign	The symbol that tells you that two sides of an equation are the same, balanced, or equal.
equivalent	Two numbers that have the same value.
fraction	A number representing part of a whole or set.
hundredths	The digit in representing $\frac{1}{100}$.
improper fraction	Any fraction in which the numerator is greater than the denominator.
least common multiple	The common multiple with the least value.
mixed number	A whole number and a fraction combined.
multiply/multiplication	The process of adding a number to itself a number of times.
multiplication sign	The symbol that tells you to multiply.
numerator	The term in a fraction that tells how many parts of a fraction.
ones	The digit representing 1.
quotient	The number that results when one number is divided by another number.
remainder	The amount left over in a division problem.
regroup/trade/exchange	The process of exchanging 10 ones for 1 ten, 10 tens for 1 hundred, 10 hundreds for 1 thousand, etc.
tenths	The digit in representing $\frac{1}{10}$.

B. Background Information

Background Information:

In this module, we focus on division with fractions and decimals. As you focus on computation of rational numbers, continue to emphasize division as partitive (i.e., equal shares) and division as quotative (i.e., measurement) because students will see these concepts within word problems.

For division of fractions, we recommend using several models of fractions to help students understand concepts related to division of fractions. We also recommend demonstrating several algorithms for division of decimals. Every student should develop efficiency with strategies for division of fractions and decimals. In the following sections, we provide examples of (1) division of fractions, (2) division of decimals with the traditional algorithm, and (3) division of decimals with the partial quotients algorithm.

C. Routines and Examples

(1) Division of Fractions*

*Most students know the *procedure* for dividing decimals but do not have *conceptual understanding* of division of fractions. Here, we provide two conceptual **Routines** (one with manipulatives and one with drawings) as well as a procedural **Routine**. Our **Example** is conceptual and uses manipulatives. Consider reading the **Example** before reading the **Routines**.

Routine

Materials:

- Module 15 Problem Sets
- Module 15 Vocabulary Cards
    - If necessary, review Vocabulary Cards before teaching
- A hands-on tool or manipulative like fraction tiles or two-color counters
    - Note that drawings can be used alongside or instead of manipulatives

ROUTINE WITH MANIPULATIVES

(Only use manipulatives with simpler problems)

Teacher	**Let's work on division. What does it mean to divide?**
Students	To share equally or measure into groups.
Teacher	**Division means to share equally or to measure into groups. Look at this problem.** (Show problem.)
Teacher	**First, I see a division sign or bracket** (point). **The division sign or bracket tells us to divide. What does the division sign or bracket mean?**
Students	To divide.
Teacher	**Let's do this problem with fraction tiles.**

	(Move fraction tiles to workspace.)
Teacher	**With division of fractions, we interpret this problem as __** (first fraction/dividend) **divided by __** (second fraction/divisor). **How do we interpret this problem?**
Students	__ divided by __.
Teacher	**When something is divided, we want to determine how many groups of the divisor we can make with the dividend. If the problem was 12 divided by 3, you would determine how many groups of 3 you could make if you had 12 of something. The same works with fractions. We'll show the dividend (or first fraction). Which fraction will we show?**
Students	The dividend or first fraction.
Teacher	**And then we'll determine how many groups of the divisor (or second fraction) we can make with the dividend. We'll determine how many groups of which fraction?**
Students	The divisor or second fraction.
Teacher	**So, let's show the dividend with the fraction tiles.** (Show dividend with fraction tiles.)
Teacher	**Now, let's find __** (divisor) **of __** (dividend)**. What's the divisor?**
Students	__.
Teacher	**Let's get out the divisor with our fraction tiles and think of it as one group.** (Show divisor with fraction tiles.)
Teacher	**Now, I want to learn how many groups of this divisor I can make with the dividend. What do I want to learn?**
Students	How many groups of the divisor we can make with the dividend.
Teacher	**To do this, I hold the divisor group under the dividend fraction tiles to see how many groups I can make. Let's see, I can make __ groups of __** (divisor). (Place divisor fraction tile group under dividend.)
Teacher	**Now, I do that again until I can't make any more groups of __** (divisor) **with the dividend.** (Place divisor fraction tile group under dividend.)
Teacher	**We're dividing by finding __** (divisor) **groups of the dividend. How are we dividing?**
Students	Finding __ (divisor) groups of the dividend.
Teacher	**We've determined __** (divisor) **groups of the dividend is __.**
Teacher	**So, __** (divisor) **groups of __** (dividend) **equals __. What's the quotient?**
Students	__.
Teacher	**__ divided by __ equals __. Let's say that together.**
Students	__ divided __ equals __.
Teacher	**So, if you have a group of __** (divisor) **and you determine you can make __ groups of __** (dividend), **__ divided by __ equals __. Let's review. What's a dividend?**
Students	The total number that will be divided.

Teacher **What's a divisor?**
Students The number of groups we will make.
Teacher **What's a quotient?**
Students The result in each group after you make groups.
Teacher **How could you explain dividing to a friend?**
Students We showed the dividend with the fraction tiles. Then, we determined how many groups of the divisor we could make with the dividend. The quotient was the number of groups we could make.

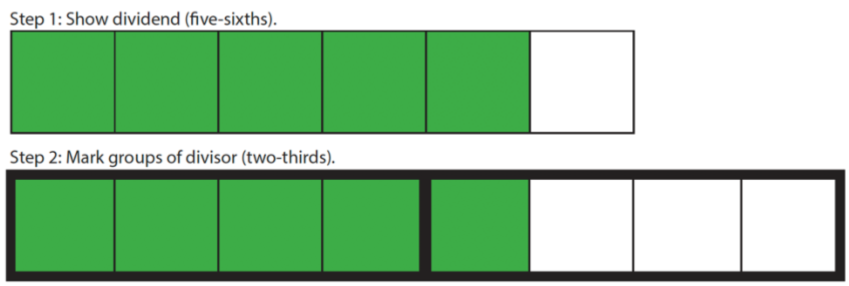
ROUTINE WITHOUT MANIPULATIVES OR DRAWINGS

Teacher **Let's work on division. What does it mean to divide?**
Students To share equally or measure into groups.
Teacher **Division means to share equally or to measure into groups. Look at this problem.**
(Show problem.)
Teacher **First, I see a division sign or bracket** (point). **The division sign or bracket tells us to divide. What does the division sign or bracket mean?**
Students To divide.
Teacher **When we divide, we divide the numerators then we divide the denominators. How do we divide?**
Students Divide the numerators then divide the denominators.
Teacher **Sometimes that's easy to do, but sometimes dividing the numerators or denominators gives us another fraction. And that gets tricky. So, often we divide fractions by using the reciprocal of the divisor. Say reciprocal with me.**
Students Reciprocal.
Teacher **With a reciprocal of a fraction, the numerator becomes the denominator and the denominator becomes the numerator. What happens with a fraction reciprocal?**
Students The numerator becomes the denominator and the denominator becomes the numerator.
Teacher **What's the reciprocal of __** (divisor)?
Students __.
Teacher **The reciprocal of __** (divisor) **is __. So, instead of dividing by __** (divisor), **we multiply by the reciprocal of the divisor. What do we do?**
Students Multiply by the reciprocal of the divisor.
Teacher **So, we multiply __** (dividend) **times __** (reciprocal of divisor). **What do we multiply?**
Students __ (dividend) times __ (reciprocal of divisor).
Teacher **First, focus on the numerators. What are the numerators in this problem?**
Students __ and __.
Teacher **What's __ times __?**

Students	__.
Teacher	**__ times __ equals __, so let's write __ as the numerator of our quotient.** (Write numerator.)
Teacher	**Let's focus on the denominators. What are the denominators in this problem?**
Students	__ and __.
Teacher	**What's __ times __?**
Students	__.
Teacher	**__ times __ equals __, so let's write __ as the denominator of our quotient.** (Write denominator.)
Teacher	**So, __** (dividend) **divided by __** (divisor) **equals __. What's the quotient?**
Students	__. (If quotient is not in simplest form, use greatest common factor to determine an equivalent fraction in simplest form.)
Teacher	**__ divided by __ equals __. Let's say that together.**
Students	__ divided by __ equals __.
Teacher	**So, if you have __** (dividend) **and you divide by __** (divisor)**, __ divided by __ equals __. Let's review. What's a dividend?**
Students	The total number that will be divided.
Teacher	**What's a divisor?**
Students	The number of groups we will make.
Teacher	**What's a quotient?**
Students	The result in each group after you make groups.
Teacher	**How could you explain dividing to a friend?**
Students	We used the reciprocal of the divisor and multiplied the dividend by the reciprocal.

Examples

$$\frac{5}{6} \div \frac{2}{3} = \frac{5}{4}$$



EXAMPLE WITH DRAWING

Teacher **Let's work on division. What does it mean to divide?**

Students To share equally or measure into groups.

Teacher **Division means to share equally or to measure into groups. Look at this problem.**
(Show problem.)

Teacher **First, I see a division sign or bracket** (point). **The division sign or bracket tells us to divide. What does the division sign or bracket mean?**

Students To divide.

Teacher **Let's do this problem by drawing. How will we do this problem?**

Students By drawing

Teacher **With division of fractions, we interpret this problem as five-sixths divided by two-thirds. How do we interpret this problem?**

Students Five-sixths divided by two-thirds.

Teacher **When something is divided, we want to determine how many groups of the divisor we can make with the dividend. With our drawing, we'll show the dividend (or first fraction). Which fraction will we draw?**

Students The dividend or first fraction.

Teacher **And then we'll determine how many groups of the divisor (or second fraction) we can make with the dividend. We'll determine how many groups of which fraction?**

Students The divisor or second fraction.

Teacher **So, let's show the dividend. I'll draw a rectangle divided into six equal parts.**
(Draw.)

Teacher **I need to shade the numerator. How many equal parts should I shade?**

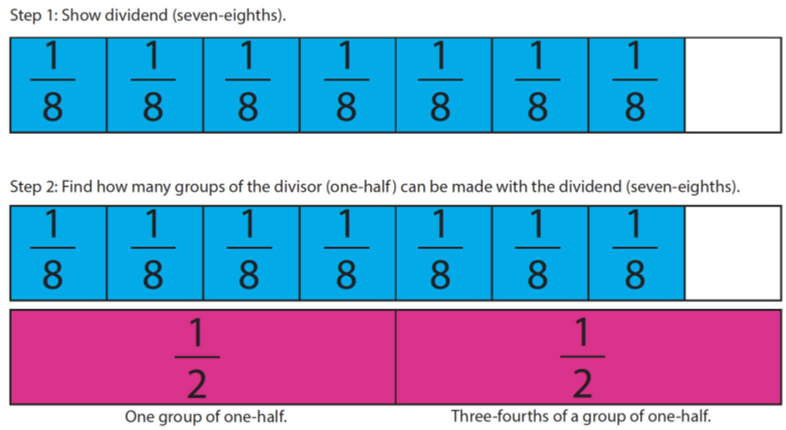
Students 5.

Teacher **So, I'll shade 5 equal parts.**
(Shade.)

Teacher	**Now, let's find two-thirds of five-sixths. What's the divisor?**
Students	Two-thirds.
Teacher	**I want to figure out how much one group of two-thirds would be when I have five-sixths. Let's see, two-thirds is equivalent to four-sixths. What's two-thirds equivalent to?**
Students	Four-sixths.
Teacher	**So, I'll draw a dark rectangle around one group of two-thirds (or four-sixths).** (Draw.)
Teacher	**I can make one full group of two-thirds. In that group of two-thirds, I see I have 1, 2, 3, 4 equal parts. So, 4 will be my new denominator. What's the new denominator?**
Students	4.
Teacher	**Let's see. I keep drawing dark rectangles around groups of two-thirds (or four-sixths) until I've used all of the five-sixth shaded parts. Let's draw another dark rectangle around the same size as the first.** (Draw.)
Teacher	**Now, with this group of two-thirds, is it a full group?**
Students	No!
Teacher	**It isn't a full group of two-thirds. How much of the group did we make? Remember, we determined 4 would be our new denominator. So, how much of this group of two-thirds is shaded?**
Students	One-fourth.
Teacher	**That's right. One-fourth of this group of two-thirds is shaded. So, when we divide five-sixths and make groups of two-thirds, we can make 1 full group of two-thirds and one-fourth of the next group of two-thirds. Our quotient is 1 and one-fourth or five-fourths. What's the quotient?**
Students	1 and one-fourth or five-fourths.
Teacher	**So, five-sixths divided by two-thirds equals five-fourths. Let's say that together.**
Students	Five-sixths divided by two-thirds equals five-fourths.
Teacher	**Let's review. What's a dividend?**
Students	The total number that will be divided.
Teacher	**What's a divisor?**
Students	The number of groups we will make.
Teacher	**What's a quotient?**
Students	The result in each group after you make groups.
Teacher	**How could you explain dividing to a friend?**
Students	We drew the dividend. Then, we determined how many groups of the divisor we could make with the dividend. The quotient was the number of groups we could make.

Example

$$\boxed{\frac{7}{8} \div \frac{1}{2} = \frac{7}{4}}$$



EXAMPLE WITH MANIPULATIVES

Teacher	**Let's work on division. What does it mean to divide?**
Students	To share equally or measure into groups.
Teacher	**Division means to share equally or to measure into groups. Look at this problem.** (Show problem.)
Teacher	**First, I see a division sign or bracket** (point). **The division sign or bracket tells us to divide. What does the division sign or bracket mean?**
Students	To divide.
Teacher	**Let's do this problem using the fraction tiles. How will we do this problem?**
Students	With the fraction tiles. (Show fraction tiles.)
Teacher	**With division of fractions, we interpret this problem as seven-eighths divided by one-half. How do we interpret this problem?**
Students	Seven-eighths divided by one-half.
Teacher	**When something is divided, we want to determine how many groups of the divisor we can make with the dividend. With our fraction tiles, we'll show the dividend (or first fraction). Which fraction will we show?**
Students	The dividend or first fraction.
Teacher	**And then we'll determine how many groups of the divisor (or second fraction) we can make with the dividend. We'll determine how many groups of which fraction?**
Students	The divisor or second fraction.
Teacher	**So, let's show the dividend. I'll show the whole and then show seven-eighths compared to the whole.** (Use fraction tiles.)
Teacher	**Now, let's find how many groups of one-half we can make with seven-eighths. What's the divisor?**

Students	One-half.
Teacher	**I want to figure out how many groups of one-half I can make if I have seven-eighths. So, I'll get out my one-half fraction tile to compare to the seven-eighths.** (Show one-half tile.)
Teacher	**Let's see how many groups of one-half I can make. I can make 1 group of one-half.** (Compare one-half tile to four one-eighth pieces.)
Teacher	**I can make one full group of one-half. How many groups?**
Students	1 full group of one-half.
Teacher	**Look closely. In this group of one-half, how many equal parts are represented?**
Students	4.
Teacher	**Yes. There are 4 equal parts in this group of one-half. That means 4 will be the denominator of our quotient. What will be the denominator?**
Students	4.
Teacher	**Let's keep going because I can make more than one group of one-half. Let's iterate (or copy) the one-half piece to see how much of the next group of one-half I can make.** (Move one-half fraction tile.)
Teacher	**Now, with this group of one-half, is it a full group?**
Students	No!
Teacher	**It isn't a full group of one-half. How much of the group did we make? Remember, we determined 4 would be our new denominator. So, how much of this group of one-half is covered by the one-half piece?**
Students	Three-fourths.
Teacher	**That's right. Three-fourths of this group of one-half is covered. So, when we divide seven-eighths and make groups of one-half, we can make 1 full group of one-half and three-fourths of the next group of one-half. Our quotient is 1 and three-fourth or seven-fourths. What's the quotient?**
Students	1 and three-fourths or seven-fourths.
Teacher	**So, seven-eighths divided by one-half equals seven-fourths. Let's say that together.**
Students	Seven-eighths divided by one-half equals seven-fourths.
Teacher	**Let's review. What's a dividend?**
Students	The total number that will be divided.
Teacher	**What's a divisor?**
Students	The number of groups we will make.
Teacher	**What's a quotient?**
Students	The result in each group after you make groups.
Teacher	**How could you explain dividing to a friend?**
Students	We showed the dividend with the fraction tiles. Then, we determined how many groups of the divisor we could make with the dividend. The quotient was the number of groups we could make.

(2) Division of Decimals with Traditional Algorithm

Routine

Materials:

- Module 15 Problem Sets
- Module 15 Vocabulary Cards
    - If necessary, review Vocabulary Cards before teaching

3-DIGIT ÷ 2-DIGIT: EXAMPLE WITHOUT MANIPULATIVES

Teacher	**Let's work on division. What does it mean to divide?**
Students	To share equally or measure into groups.
Teacher	**Division means to share equally or to measure into groups. Look at this problem.** (Show problem.)
Teacher	**First, I see a division bracket** (point). **The division bracket tells us to divide. What does the division bracket mean?**
Students	To divide.
Teacher	**Let's do this problem with our pencil. First, when I see a problem like this that requires computation, I like to draw vertical lines to separate the ones from the tenths and the tenths from the hundredths. Let's draw a vertical line between each of the columns in the dividend.** (Draw vertical lines to separate place value columns.)
Teacher	**Now, we start by dividing the dividend by the divisor. What's our dividend?**
Students	__.
Teacher	**And we'll divide the dividend by the divisor. What's the divisor?**
Students	__.
Teacher	**When we divide using this method, for each place value in the dividend, the first thing we do is divide. If we can divide, then we multiply, subtract, and then bring in the next place value. So, the pattern is: divide, multiply, subtract, bring in. Say that with me.**
Students	Divide, multiply, subtract, bring in.
Teacher	**And we keep repeating that pattern until we have solved the problem. Let's see how it works. Are you ready?**
Students	Yes!
Teacher	**Okay, so we start with the greatest place value of the dividend. Where do we start?**
Students	Greatest place value of the dividend.
Teacher	**In this problem, the greatest place value of the dividend is __. What number?**
Students	__.
Teacher	**How many groups of __** (divisor) **can we make with __?**

Students	We can't make any groups of __.
Teacher	**We can't make a group of __** (divisor)**. So, now we bring in the __** (next place value in dividend) **to make __. I think how many groups of __ can we make if we have __** (divisor)**?**
Students	We can make __ groups of __ (divisor).
Teacher	**We can make __ groups. So, let's write __ above the division bracket.** (Write.)
Teacher	**So, now let's multiply __ times __** (divisor)**. What's __ times __?**
Students	__.
Teacher	**Let's write that product of __ below the __ in the dividend.** (Write.)
Teacher	**Now, let's write a minus sign and an equal line to help us subtract __ from __. What sign?**
Students	Minus sign.
Teacher	**What do we subtract?**
Students	__ minus __.
Teacher	**What's __ minus __?**
Students	__.
Teacher	**Let's write the difference here under the equal line.** (Write.)
Teacher	**Now, we bring in the next digit of the dividend to our difference. I like to show this by drawing an arrow from the __ and rewriting the __ next to __.** (Draw arrow and write.)
Teacher	**When I bring in the __, __ now becomes __. This is our new dividend. What's our new dividend?**
Students	__.
Teacher	**So, we followed the steps of division: divide, multiply, subtract, bring in. Say that with me.**
Students	Divide, multiply, subtract, bring in.
Teacher	**But the problem isn't finished. Let's follow the steps again: divide, multiply, subtract, bring in. What do we do?**
Students	Divide, multiply, subtract, bring in.
Teacher	**How many groups of __** (divisor) **can we make with our new dividend of __?**
Students	__ groups.
Teacher	**We can make __ groups. So, let's write __ above the division bracket.** (Write.)
Teacher	**So, let's multiply. What's __ times __** (divisor)**?**
Students	__.
Teacher	**Let's write __ below the __**. (Write.)
Teacher	**Now, let's write a minus sign and an equal line. What sign?**
Students	Minus sign.
Teacher	**And let's subtract __ minus __. What do we subtract?**
Students	__ minus __.

Teacher	**What’s __ minus __?**
Students	__.
Teacher	**Let’s write the difference here under the equal line.** (Write.)
Teacher	**Now, do we have any remaining?**
Students	Yes!
Teacher	**Just like before, we have to bring in a number to keep dividing. I’ll bring in the next digit from the dividend.** (Draw arrow and write.)
Teacher	**When I bring in the __, __ now becomes __. This is our new dividend. What’s our new dividend?**
Students	__.
Teacher	**Let’s follow the steps again: divide, multiply, subtract, bring in. What do we do?**
Students	Divide, multiply, subtract, bring in.
Teacher	**How many groups of __** (divisor) **can we make with our new dividend of __?**
Students	__ groups.
Teacher	**We can make __ groups. So, let’s write __ above the division bracket.** (Write.)
Teacher	**So, let’s multiply. What’s __ times __** (divisor)**?**
Students	__.
Teacher	**Let’s write __ below the __.** (Write.)
Teacher	**Now, let’s write a minus sign and an equal line. What sign?**
Students	Minus sign.
Teacher	**And let’s subtract __ minus __. What do we subtract?**
Students	__ minus __.
Teacher	**What’s __ minus __?**
Students	__.
Teacher	**Let’s write the difference here under the equal line.** (Write.)
Teacher	**Now, do we have any remaining?**
Students	No!
Teacher	**Now, we seem finished but we’re not. In this problem, we divided decimals. So, we have to place the decimal point in the quotient. What do we have to place in the product?**
Students	A decimal point.
Teacher	**To place the decimal point, we determine the number of decimal places in the dividend and divisor. Let’s see. The dividend had __ decimal places. The divisor also had __ decimal places. What’s __ plus __?**
Students	_.
Teacher	**So, in the quotient, we need to put in __ decimal places starting from the least place value (or the right). That means I’ll place a decimal point between the __ and __.**

Teacher	**So, what's the quotient?**
Students	__.
Teacher	**The quotient is __. So, __** (dividend) **divided by __** (divisor) **equals __. Say that with me.**
Students	__ divided by __ equals __.
Teacher	**So, if you have __ and divide by __, the quotient is __. Let's review. What's a dividend?**
Students	The total number that will be divided.
Teacher	**What's a divisor?**
Students	The number of groups we will make.
Teacher	**What's a quotient?**
Students	The result in each group after you equally share or measure groups.
Teacher	**How could you explain dividing to a friend?**
Students	We asked ourselves about how many groups we can make with the divisor from the dividend. The number of groups is the quotient.

Example

```
     0.788
  5) 3.94 0
   – 3 5↓
       4 4
     – 4 0↓
         4 0
       – 4 0
           0
```

3-DIGIT ÷ 1-DIGIT: EXAMPLE WITHOUT MANIPULATIVES

Teacher	**Let's work on division. What does it mean to divide?**
Students	To share equally or measure into groups.
Teacher	**Division means to share equally or to measure into groups. Look at this problem.** (Show problem.)
Teacher	**First, I see a division bracket** (point). **The division bracket tells us to divide. What does the division bracket mean?**
Students	To divide.
Teacher	**Let's do this problem with our pencil. First, when I see a problem like this that requires computation, I like to draw vertical lines to separate the ones from the tenths and the tenths from the hundredths. Let's draw a vertical line between each of the columns in the dividend.** (Draw vertical lines to separate place value columns.)
Teacher	**Now, we start by dividing the dividend by the divisor. What's our dividend?**

Students	3.94.
Teacher	**And we'll divide the dividend by the divisor. What's the divisor?**
Students	5.
Teacher	**When we divide using this method, for each place value in the dividend, the first thing we do is divide. If we can divide, then we multiply, subtract, and then bring in the next place value. So, the pattern is: divide, multiply, subtract, bring in. Say that with me.**
Students	Divide, multiply, subtract, bring in.
Teacher	**And we keep repeating that pattern until we have solved the problem. Let's see how it works. Are you ready?**
Students	Yes!
Teacher	**Okay, so we start with the greatest place value of the dividend. Where do we start?**
Students	Greatest place value of the dividend.
Teacher	**In this problem, the greatest place value of the dividend is 3. What number?**
Students	3.
Teacher	**How many groups of 5 can we make with 3?**
Students	We can't make any groups of 5 if we have 3.
Teacher	**We can't make a group of 5. So, now we bring in the 9 to make 39. I think how many groups of 39 can we make if we have 5?**
Students	We can make 7 groups of 5.
Teacher	**We can make 7 groups. So, let's write 7 above the division bracket in the tenths column.** (Write 7.)
Teacher	**So, now let's multiply 7 times 5. What's 7 times 5?**
Students	35.
Teacher	**Let's write that product of 35 below the 39 in the dividend.** (Write 35.)
Teacher	**Now, let's write a minus sign and an equal line to help us subtract 35 from 39. What sign?**
Students	Minus sign.
Teacher	**What do we subtract?**
Students	39 minus 35.
Teacher	**What's 39 minus 35?**
Students	4.
Teacher	**Let's write the difference here under the equal line.** (Write 4.)
Teacher	**Now, we bring in the hundredth to our difference. I like to show this by drawing an arrow from the 4 and rewriting the 4 next to 4.** (Draw arrow and write 4.)
Teacher	**When I bring in the 4, 4 now becomes 44. This is our new dividend. What's our new dividend?**
Students	44.

Teacher	**So, we followed the steps of division: divide, multiply, subtract, bring in. Say that with me.**
Students	Divide, multiply, subtract, bring in.
Teacher	**But the problem isn't finished. Let's follow the steps again: divide, multiply, subtract, bring in. What do we do?**
Students	Divide, multiply, subtract, bring in.
Teacher	**How many groups of 5 can we make with our new dividend of 44?**
Students	8 groups.
Teacher	**We can make 8 groups. So, let's write 8 above the division bracket in the hundredths column.** (Write 8.)
Teacher	**So, let's multiply. What's 8 times 5?**
Students	40.
Teacher	**Let's write 40 below the 44**. (Write 40.)
Teacher	**Now, let's write a minus sign and an equal line. What sign?**
Students	Minus sign.
Teacher	**And let's subtract 44 minus 40. What do we subtract?**
Students	44 minus 40.
Teacher	**What's 44 minus 40?**
Students	4.
Teacher	**Let's write the difference here under the equal line.** (Write 4.)
Teacher	**Now, do we have any remaining?**
Students	Yes!
Teacher	**Just like before, we have to bring in a number to keep dividing. This time, I'll bring in from the hundredths place. Is there a number written in the hundredths place?**
Students	No.
Teacher	**There is no number written there. But what number is in the thousandths place just holding place value?**
Students	Zero.
Teacher	**Yes, we assume a zero is in the thousandths place. So, I'll bring in a 0. I like to show this by drawing an arrow from the assumed 0 and writing the 0 next to 4.** (Draw arrow and write 0.)
Teacher	**When I bring in the 0, 4 now becomes 40. This is our new dividend. What's our new dividend?**
Students	40.
Teacher	**Let's follow the steps again: divide, multiply, subtract, bring in. What do we do?**
Students	Divide, multiply, subtract, bring in.
Teacher	**How many groups of 5 can we make with our new dividend of 40?**
Students	8 groups.

Teacher	**We can make 8 groups. So, let's write 8 above the division bracket in the thousandths column.** (Write 8.)
Teacher	**So, let's multiply. What's 8 times 5?**
Students	40.
Teacher	**Let's write 40 below the 40**. (Write 40.)
Teacher	**Now, let's write a minus sign and an equal line. What sign?**
Students	Minus sign.
Teacher	**And let's subtract 40 minus 40. What do we subtract?**
Students	40 minus 40.
Teacher	**What's 40 minus 40?**
Students	0.
Teacher	**Let's write the difference here under the equal line.** (Write 0.)
Teacher	**Now, do we have any remaining?**
Students	No!
Teacher	**Now, we seem finished but we're not. In this problem, we divided decimals. So, we have to place the decimal point in the quotient. What do we have to place in the product?**
Students	A decimal point.
Teacher	**To place the decimal point, we determine the number of decimal places in the dividend and divisor. Let's see. The dividend had 3 decimal places. The divisor had 0 decimal places. What's 3 plus 0?**
Students	3.
Teacher	**So, in the quotient, we need to put in 3 decimal places starting from the least place value (or the right). That means I'll place a decimal point between the 0 and 7.**
Teacher	**So, what's the quotient?**
Students	0.788.
Teacher	**The quotient is 0.788. So, 3.94 divided by 5 equals 0.788. Say that with me.**
Students	3.94 divided by 5 equals 0.788.
Teacher	**So, if you have 3.94 and divide by 5**, **the quotient is 0.788. Let's review. What's a dividend?**
Students	The total number that will be divided.
Teacher	**What's a divisor?**
Students	The number of groups we will make.
Teacher	**What's a quotient?**
Students	The result in each group after you equally share or measure groups.
Teacher	**How could you explain dividing to a friend?**
Students	We asked ourselves about how many groups we can make with the divisor from the dividend. The number of groups is the quotient.

(3) Division with Partial Quotients Algorithm*

*For clarity, read **Example** before using **Routines**.

Routine

Materials:

- Module 15 Problem Sets
- Module 15 Vocabulary Cards
  - If necessary, review Vocabulary Cards before teaching

3-DIGIT ÷ 2-DIGIT: ROUTINE WITHOUT MANIPULATIVES

Teacher	**Let's work on division. What does it mean to divide?**
Students	To share equally or measure into groups.
Teacher	**Division means to share equally or to measure into groups. Look at this problem.** (Show problem.)
Teacher	**First, I see a division bracket** (point). **The division bracket tells us to divide. What does the division bracket mean?**
Students	To divide.
Teacher	**Let's do this problem with our pencil, and let's use the partial quotients strategy. If I want to use the partial quotients strategy, I first draw a vertical line down from the end of the division bracket.** (Draw vertical line from end of division bracket.)
Teacher	**With the partial quotients strategy, we divide the dividend a few different times. Each time we create a partial quotient. At the end, we add the partial quotients to determine the final quotient. Which strategy are we using again?**
Students	Partial quotients.
Teacher	**Now, we start by dividing the dividend by the divisor. What's our dividend?**
Students	__.
Teacher	**And we'll divide the dividend by the divisor. What's the divisor?**
Students	__.
Teacher	**When we divide with decimals, let's ignore all the decimals for now. We'll interpret this as __divided by __. We'll bring back the decimals at the end. What will we ignore for now?**
Students	Decimals.
Teacher	**I don't know exactly how many groups of __** (divisor) **I can make with __** (dividend)**, so the partial quotients strategy can be used with computation that I do know. Which strategy are we using?**
Students	Partial quotients.
Teacher	**How many groups of __** (divisor) **can we make with __** (dividend)**?**
Students	I'm not sure.
Teacher	**I don't know the exact answer either, so I'll use a partial quotient to start solving this problem. I know that __** (friendly number) **groups of __** (divisor)

	would be __ (friendly number times divisor product)**, so I'll write __** (product) **under the __** (dividend)**. I'll also write __** (partial quotient) **to the right of the vertical line. __ is one of my partial quotients.** (Write.)
Teacher	**Now, I'll subtract __** (product) **from the dividend of __ to determine a new dividend. I write a minus sign and an equal line.** (Write minus sign and equal line.)
Teacher	**__** (dividend) **minus __** (product) **equals what?**
Students	__.
Teacher	**Let's write the difference of __ below the equal line.** (Write.)
Teacher	**Now, how many groups of __** (divisor) **can we make with __** (new dividend)**?**
Students	I'm not sure.
Teacher	**Again, I don't know the exact answer either, so I'll use a partial quotient. I know that __** (friendly number) **groups of __** (divisor) **would be __. I'll write __** (product) **under the __** (new dividend)**. I'll also write __** (partial quotient) **to the right of the vertical line. __ is one of my partial quotients.** (Write.)
Teacher	**Now, I'll subtract __** (product) **from the dividend of __** (new dividend) **to determine a new dividend. I write a minus sign and an equal line.** (Write minus sign and equal line.)
Teacher	**__** (new dividend) **minus __** (product) **equals what?**
Students	__.
Teacher	**Let's write the difference of __ below the equal line.** (Write.)
Teacher	**Now, how many groups of __** (divisor) **can we make with __?**
Students	__.
Teacher	**Yes! I know that __** (friendly number) **groups of __** (divisor) **would be __. I'll write __ under the __** (new dividend)**. I'll also write __** (partial quotient) **to the right of the vertical line. __ is one of my partial quotients.** (Write.)
Teacher	**Now, I'll subtract __ from the dividend of __ to determine a new dividend. I write a minus sign and an equal line.** (Write minus sign and equal line.)
Teacher	**__ minus __ equals what?**
Students	__.
Teacher	**Let's write the difference of __ below the equal line.** (Write.)
Teacher	**This is our new dividend. Can we make any more groups of __** (divisor)**?**
Students	No!
Teacher	**We can't make any more groups of __** (divisor)**, so let's determine our quotient. We do this by adding the partial quotients together. How do we determine the quotient?**
Students	Add the partial quotients together.

Teacher	**Let’s write a plus sign and equal line.** (Write plus sign and equal line.)
Teacher	**What’s __ plus __ plus …?**
Students	__.
Teacher	**Let’s write the sum of the partial quotients below the equal line.** (Write.)
Teacher	**We also could write the quotient above the division bracket.** (Write.)
Teacher	**What’s the quotient?**
Students	__.
Teacher	**Now, we seem finished but we’re not. In this problem, we divided decimals. So, we have to place the decimal point in the quotient. What do we have to place in the quotient?**
Students	A decimal point.
Teacher	**To place the decimal point, we determine the number of decimal places in the dividend and divisor. Let’s see. The dividend had __ decimal places. The divisor also had __ decimal places. What’s __ plus __?**
Students	__.
Teacher	**So, in the quotient, we need to put in __ decimal places starting from the greatest place value of the quotient. That means I’ll place a decimal point __.** (Write decimal point.)
Teacher	**So, what’s the quotient?**
Students	__.
Teacher	**So, __ divided by __ equals __. Let’s say that together.**
Students	__ divided by __ equals __.
Teacher	**Let’s say it together again.**
Students	__ divided by __ equals __.
Teacher	**Let’s review. What’s a dividend?**
Students	The total number that will be divided.
Teacher	**What’s a divisor?**
Students	The number of groups we will make.
Teacher	**What’s a quotient?**
Students	The result in each group after you equally share or measure groups.
Teacher	**How could you explain partial quotients to a friend?**
Students	We kept asking how many groups of the divisor we could make with the dividend. We didn’t know the exact answer, so we used computation we did know as partial quotients. At the end, we added the partial quotients for the final quotient.

Example

```
      31.0
2.4 ) 74.4
    – 480  | 20
      264
    – 240  | 10
       24
    –  24  | + 1
           | 31
```

3-DIGIT ÷ 2-DIGIT: EXAMPLE WITHOUT MANIPULATIVES

Teacher	**Let's work on division. What does it mean to divide?**
Students	To share equally or measure into groups.
Teacher	**Division means to share equally or to measure into groups. Look at this problem.** (Show problem.)
Teacher	**First, I see a division bracket** (point). **The division bracket tells us to divide. What does the division bracket mean?**
Students	To divide.
Teacher	**Let's do this problem with our pencil, and let's use the partial quotients strategy. If I want to use the partial quotients strategy, I first draw a vertical line down from the end of the division bracket.** (Draw vertical line from end of division bracket.)
Teacher	**With the partial quotients strategy, we divide the dividend a few different times. Each time we create a partial quotient. At the end, we add the partial quotients to determine the final quotient. Which strategy are we using again?**
Students	Partial quotients.
Teacher	**Now, we start by dividing the dividend by the divisor. What's our dividend?**
Students	74.4.
Teacher	**And we'll divide the dividend by the divisor. What's the divisor?**
Students	2.4.
Teacher	**When we divide with decimals, let's ignore all the decimals for now. We'll interpret this as 744 divided by 24. We'll bring back the decimals at the end. What will we ignore for now?**
Students	Decimals.
Teacher	**I don't know exactly how many groups of 24 I can make with 744, so the partial quotients strategy can be used with computation that I do know. Which strategy are we using?**
Students	Partial quotients.
Teacher	**How many groups of 24 can we make with 744?**
Students	I'm not sure.

Teacher	**I don't know the exact answer either, so I'll use a partial quotient to start solving this problem. I know that 20 groups of 24 would be 480, so I'll write 480 under the 744. I'll also write 20 to the right of the vertical line. 20 is one of my partial quotients.** (Write 480 and 20.)
Teacher	**Now, I'll subtract 480 from the dividend of 744 to determine a new dividend. I write a minus sign and an equal line.** (Write minus sign and equal line.)
Teacher	**744 minus 480 equals what?**
Students	264.
Teacher	**Let's write the difference of 264 below the equal line.** (Write 264.)
Teacher	**Now, how many groups of 24 can we make with 264?**
Students	I'm not sure.
Teacher	**Again, I don't know the exact answer either, so I'll use a partial quotient. I know that 10 groups of 24 would be 240. I'm using computation that's easier for me to do – so I like to think about 20 groups of 24 or 10 groups of 24. So, 10 groups of 24 equals 240. I'll write 240 under the 264. I'll also write 10 to the right of the vertical line. 10 is one of my partial quotients.** (Write 240 and 10.)
Teacher	**Now, I'll subtract 240 from the dividend of 264 to determine a new dividend. I write a minus sign and an equal line.** (Write minus sign and equal line.)
Teacher	**264 minus 240 equals what?**
Students	24.
Teacher	**Let's write the difference of 24 below the equal line.** (Write 24.)
Teacher	**Now, how many groups of 24 can we make with 24?**
Students	1!
Teacher	**Yes! I know that 1 group of 24 would be 24. I'll write 24 under the 24. I'll also write 1 to the right of the vertical line. 1 is one of my partial quotients.** (Write 24 and 1.)
Teacher	**Now, I'll subtract 24 from the dividend of 24 to determine a new dividend. I write a minus sign and an equal line.** (Write minus sign and equal line.)
Teacher	**24 minus 24 equals what?**
Students	0.
Teacher	**Let's write the difference of 0 below the equal line.** (Write 0.)
Teacher	**This 0 is our new dividend. Can we make any more groups of 24?**
Students	No!
Teacher	**We can't make any more groups of 24, so let's determine our quotient. We do this by adding the partial quotients together. How do we determine the quotient?**

Students	Add the partial quotients together.
Teacher	**Let's write a plus sign and equal line.** (Write plus sign and equal line.)
Teacher	**What's 20 plus 10 plus 1?**
Students	31.
Teacher	**Let's write the sum of the partial quotients below the equal line.** (Write 31.)
Teacher	**We also could write the quotient above the division bracket.** (Write 31.)
Teacher	**What's the quotient?**
Students	31.
Teacher	**Now, we seem finished but we're not. In this problem, we divided decimals. So, we have to place the decimal point in the quotient. What do we have to place in the quotient?**
Students	A decimal point.
Teacher	**To place the decimal point, we determine the number of decimal places in the dividend and divisor. Let's see. The dividend had 1 decimal place. The divisor also had 1 decimal place. What's 1 plus 1?**
Students	2.
Teacher	**So, in the quotient, we need to put in 2 decimal places starting from the greatest place value of the quotient. That means I'll place a decimal point after the 31. I would write 31.0 if I wanted to do so.** (Write decimal point.)
Teacher	**So, what's the quotient?**
Students	31.0.
Teacher	**So, 74.4 divided by 2.4 equals 31.0. Let's say that together.**
Students	74.4 divided by 2.4 equals 31.0.
Teacher	**Let's say it together again.**
Students	74.4 divided by 2.4 equals 31.0.
Teacher	**Let's review. What's a dividend?**
Students	The total number that will be divided.
Teacher	**What's a divisor?**
Students	The number of groups we will make.
Teacher	**What's a quotient?**
Students	The result in each group after you equally share or measure groups.
Teacher	**How could you explain partial quotients to a friend?**
Students	We kept asking how many groups of 24 we could make with the dividend. We didn't know the exact answer, so we used computation we did know as partial quotients. At the end, we added the partial quotients for the final quotient.

D. Problems for Use During Instruction

See Module 15 Problem Sets.

E. Vocabulary Cards for Use During Instruction

See Module 15 Vocabulary Cards.

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Module 15:
Division of Rational Numbers

Problem Sets

A.

$$\frac{2}{3} \div \frac{1}{2} =$$

A. $$\frac{5}{8} \div \frac{1}{2} =$$

A.

$$\frac{3}{4} \div \frac{1}{2} =$$

A.

$$\frac{3}{5} \div \frac{2}{3} =$$

A. $\frac{1}{6} \div \frac{2}{3} =$

A.

$$\frac{8}{10} \div \frac{4}{5} =$$

A.

$$\frac{1}{4} \div \frac{1}{2} =$$

A.

$$\frac{1}{5} \div \frac{2}{5} =$$

A. $\frac{4}{8} \div \frac{1}{4} =$

A.

$$\frac{1}{2} \div \frac{3}{4} =$$

A. $\frac{8}{12} \div \frac{1}{6} =$

A.

$$\frac{4}{5} \div \frac{1}{2} =$$

A.

$$\frac{1}{5} \div \frac{2}{3} =$$

A.

$$\frac{1}{3} \div \frac{3}{4} =$$

A.

$$\frac{5}{6} \div \frac{1}{8} =$$

A. $\frac{4}{5} \div \frac{3}{4} =$

A.

$$\frac{4}{6} \div \frac{1}{2} =$$

A.

$$\frac{7}{10} \div \frac{1}{4} =$$

A.

$$\frac{8}{10} \div \frac{3}{4} =$$

A. $\frac{5}{6} \div \frac{1}{3} =$

A.

$$\frac{4}{5} \div \frac{1}{6} =$$

A.

$$\frac{6}{8} \div \frac{1}{4} =$$

A.

$$\frac{5}{8} \div \frac{2}{3} =$$

A.

$$\frac{5}{10} \div \frac{1}{4} =$$

A.

$$\frac{8}{10} \div \frac{2}{5} =$$

A.

$$\frac{6}{9} \div \frac{1}{3} =$$

A.

$$\frac{1}{4} \div \frac{1}{2} =$$

A.

$$\frac{9}{10} \div \frac{2}{5} =$$

A.

$$\frac{2}{5} \div \frac{1}{2} =$$

A. $$\frac{5}{6} \div \frac{1}{2} =$$

B.

$$\frac{7}{3} \div \frac{1}{2} =$$

B.

$$\frac{6}{5} \div \frac{1}{3} =$$

B.

$$\frac{8}{3} \div \frac{1}{8} =$$

B. $\frac{12}{4} \div \frac{2}{3} =$

B. $\dfrac{6}{4} \div \dfrac{1}{8} =$

B.

$$\frac{9}{8} \div \frac{1}{5} =$$

B. $\frac{16}{3} \div \frac{1}{8} =$

B.

$$\frac{6}{4} \div \frac{1}{6} =$$

B.

$$\frac{9}{3} \div \frac{4}{5} =$$

B.

$$\frac{8}{3} \div \frac{2}{3} =$$

B.

$$\frac{6}{5} \div \frac{2}{3} =$$

B.

$$\frac{4}{3} \div \frac{2}{4} =$$

B. $\dfrac{7}{5} \div \dfrac{1}{2} =$

B. $\dfrac{5}{2} \div \dfrac{2}{3} =$

B.

$$\frac{4}{3} \div \frac{3}{8} =$$

c.

$$4\frac{1}{4} \div \frac{1}{2} =$$

c.

$$3\frac{3}{4} \div \frac{1}{6} =$$

c.

$$6\frac{1}{2} \div \frac{1}{2} =$$

c.

$$2\frac{3}{5} \div \frac{2}{5} =$$

c.

$$1\frac{7}{8} \div \frac{3}{4} =$$

c.

$$4\frac{4}{5} \div \frac{4}{5} =$$

c.

$$5\frac{2}{3} \div \frac{1}{5} =$$

c.

$$6\frac{1}{2} \div \frac{1}{4} =$$

c.

$$4\frac{2}{5} \div \frac{1}{3} =$$

c. $3\frac{1}{6} \div \frac{2}{3} =$

c.

$$2\frac{1}{6} \div \frac{1}{8} =$$

c.

$$2\frac{1}{5} \div \frac{1}{2} =$$

c.

$$8\frac{5}{6} \div \frac{5}{6} =$$

c.

$$2\frac{1}{4} \div \frac{1}{2} =$$

c.

$$9\frac{5}{6} \div \frac{1}{3} =$$

D.

$$0.3\overline{)2.4}$$

D.

$$0.5\overline{)4.5}$$

D.

$0.7\overline{)9.1}$

D.

$$5\overline{)7.0}$$

D.

$$0.1 \overline{)\,0.8}$$

D.

$$0.6\overline{)4.2}$$

D.

$$0.8\overline{)7.2}$$

D.

$$0.1\overline{)3.6}$$

D.

$$0.2\overline{)2.8}$$

D.

$$0.9\overline{)5.4}$$

D.

$$0.4\overline{)9.6}$$

D.

$$3\overline{)3.6}$$

D.

$$0.7\overline{)7.7}$$

D.

$$0.6\overline{)4.8}$$

D.

$$0.3\overline{)5.1}$$

D.

$$8\overline{)7.2}$$

D.

$$0.8\overline{)9.6}$$

D.

$$0.4\overline{)7.2}$$

D.

$$0.5\overline{)9.5}$$

D.

$$0.2\overline{)6.4}$$

E.

$$6\overline{)8.64}$$

E.

$$3\overline{)3.93}$$

E.

$$8\overline{)4.32}$$

E.

$$4\overline{)8.84}$$

E.

$$5\overline{)7.20}$$

E.

$$9\overline{)9.09}$$

E.

$$7\overline{)5.25}$$

E.

$$8\overline{)2.88}$$

E.

$$6\overline{)9.00}$$

E.

$$3\overline{)4.38}$$

E.

$$0.04\overline{)16}$$

E.

$$3\overline{)\,.24}$$

E.

$$5 \overline{)3.65}$$

E.

$$4\overline{)12.08}$$

E.

$$.08\overline{\smash{)}.64}$$

E.

$$0.09\overline{)10.89}$$

E.

$$0.16\overline{)5.67}$$

E.

$$.45\overline{)14.85}$$

E.

$$6.2\overline{)23.25}$$

E.

$$3\overline{)96.3}$$

F.

$$3.6\overline{)27.68}$$

F.

$$4.18\overline{)41.80}$$

F.

$$0.8\overline{)1.68}$$

F.

$$4.2\overline{)14.28}$$

F.

$$3.3\overline{)20.46}$$

F.

$$2.3\overline{)19.32}$$

F.

$$1.8\overline{)15.48}$$

F.

$$4.8 \overline{)5.28}$$

F.

$$1.9\overline{)93.1}$$

F.

$$5.1\overline{)19.38}$$

F.

$$0.9\overline{)9.54}$$

F.

$$7.9\overline{)51.35}$$

F.

$$2.1 \overline{\smash{)}31.5}$$

F.

$$6.8\overline{)88.4}$$

F.

$$6.1\overline{\smash{)}12.2}$$

F.

$$4.5\overline{)17.1}$$

F.

$$7.8\overline{)31.98}$$

F.

$$3.1 \overline{\smash{\big)}\,24.18}$$

F.

$$1.7\overline{)54.23}$$

F.

$$3.7\overline{)92.5}$$

F.

$$3.8\overline{)83.6}$$

F.

$$0.3\overline{)\,79.5}$$

F.

$$3.8\overline{)47.5}$$

F.

$$5.1\overline{)75.99}$$

F.

$$2.1\overline{)1.47}$$

F.

$$4.8\overline{\smash{)}47.07}$$

F.

$$4.3\overline{)36.12}$$

F.

$$1.8\overline{)\,7.2\,}$$

F.

$$8.4\overline{)24.36}$$

F.

$$1.9\overline{)81.89}$$

G.

$$2\overline{)1.09}$$

G.

$$6\overline{)7.18}$$

G.

$$9\overline{)2.69}$$

G.

$$4\overline{)9.83}$$

G.

$$3\overline{)8.41}$$

G.

$$9.13\overline{)9.92}$$

G.

$$5.99\overline{)41.9}$$

G.

$$6.04\overline{)75.96}$$

G.

$$6.21\overline{)6.99}$$

G.

$$8.64 \overline{)\,47.4}$$

Module 15:
Division of Rational Numbers

Vocabulary Cards

algorithm
computation
decimal
denominator
divide/division
dividend
division sign
divisor
equal groups
equal sign
fractions
hundredths
improper fraction
least common multiple
mixed number
numerator
ones
quotient
reciprocal
remainder
regroup/trade/exchange
tenths

algorithm

A procedure or description of steps that can be used to solve a problem.

computation

The action used to solve a problem.

decimal

A number based on powers of ten.

34.107

tens ones tenths hundredths thousandths

---

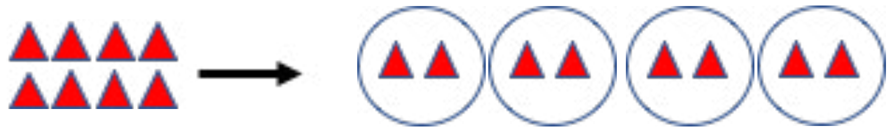
denominator

The term in a fraction that tells the number of equal parts in a whole.

$2 / 3$ $\qquad \frac{2}{3}$ **In these fractions, 3 is the denominator.**

divide/division

To separate into equal groups.



$8 \div 4 = 2$

---

dividend

The number that is to be divided in a division problem.

$16 \div 8 = 2$

16 is the dividend

division sign

The symbol that tells you to divide.

$$16 \div 8 = 2$$

÷ is the division sign

---

divisor

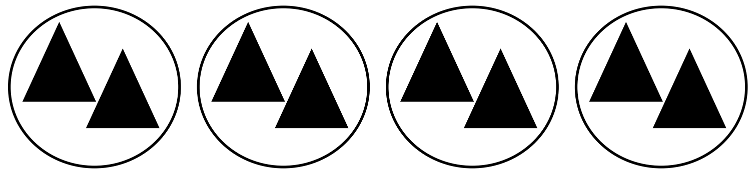
The number that the dividend is divided by.

$$16 \div 8 = 2$$

8 is the divisor

equal groups

Groups with the same number of objects or items in each group.



equal sign

The symbol that tells you that two sides of an equation are the same, balanced, or equal.

$$16 \div 8 = 2$$

= is the equal sign

fraction

A number representing part of a whole or set.

$$\frac{3}{6} \qquad \frac{10}{12} \qquad \frac{8}{3}$$

---

hundredths

The digit in representing $\frac{1}{100}$.

In the number 4.23, 3 is in the hundredths place.

improper fraction

Any fraction in which the numerator is greater than the denominator.

$$\frac{9}{4} \qquad \frac{17}{12} \qquad \frac{10}{3}$$

---

least common multiple

The common multiple with the least value.

6: 6, 12, 18, 24, 30

8: 8, 16, 24, 32, 40

With multiples of 6 and 8, the least common multiple is 24.

mixed number

A whole number and a fraction combined.

$$1\frac{1}{6} \qquad 4\frac{5}{12} \qquad 12\frac{4}{3}$$

---

numerator

The term in a fraction that tells how many parts of a fraction.

$$2 / 3 \qquad \frac{2}{3}$$

In these fractions, 2 is the numerator.

ones

The digit representing 1.

In the number 4.23, 4 is in the ones place.

---

quotient

The number that results when one number is divided by another number.

$$16 \div 8 = 2$$

2 is the quotient

reciprocal

The reciprocal of a number is 1 divided by that number.

original number	reciprocal
$\frac{4}{9}$	$\frac{9}{4}$

remainder

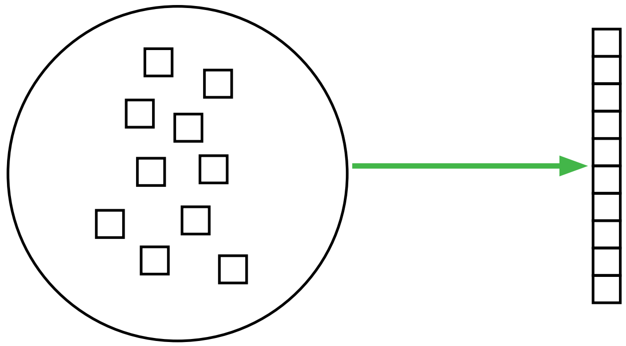
The amount left over in a division problem.

$$\begin{array}{r}
4\ \text{R}\,2 \\
20\overline{)\,82} \\
-\ 80 \\
\hline
2
\end{array}$$

2 is the remainder

regroup/trade/exchange

The process of exchanging 10 ones for 1 ten, 10 tens for 1 hundred, 10 hundreds for 1 thousand, etc.



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tenths

The digit in representing $\frac{1}{10}$.

In the number 4.2 3, 2 is in the tenths place.