



Instructional Routines for Mathematics Intervention

The purpose of these mathematics instructional routines is to provide educators with materials to use when providing intervention to students who experience difficulty with mathematics. The routines address content included in the grades 2-8 Texas Essential Knowledge and Skills (TEKS). There are 23 modules that include routines and examples – each focused on different mathematical content. Each of the 23 modules include vocabulary cards and problem sets to use during instruction. These materials are intended to be implemented explicitly with the aim of improving mathematics outcomes for students.

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Instructional Routines for Mathematics Intervention

MODULE 6

Addition of Rational Numbers



Module 6: Addition of Rational Numbers

Mathematics Routines

A. Important Vocabulary with Definitions

Term	Definition
add/addition	To put amounts together to find the sum or to increase a set.
addend	Any numbers that are added together.
algorithm	A procedure or description of steps that can be used to solve a problem.
computation	The action used to solve a problem.
decimal	A number based on powers of ten.
denominator	The term in a fraction that tells the number of equal parts in a whole.
equal sign	The symbol that tells you that two sides of an equation are the same, balanced, or equal.
equivalent	Two numbers that have the same value.
fraction	A number representing part of a whole or set.
hundredths	The digit in representing $\frac{1}{100}$.
improper fraction	Any fraction in which the numerator is greater than or equal to the denominator.
join	To add to an existing set.
least common multiple	The common multiple with the least value.
mixed number	A whole number and a fraction combined.
multiple	The product of a number and any integer.
numerator	The term in a fraction that tells how many parts of a fraction.
ones	The digit representing 1.
plus sign	The symbol that tells you to add.
regroup/trade/exchange	The process of exchanging 10 ones for 1 ten, 10 tens for 1 hundred, 10 hundreds for 1 thousand, etc.
sum	The result of adding two or more numbers.
tenths	The digit in representing $\frac{1}{10}$.
together	To combine sets or numbers.

B. Background Information

In this module, we focus on addition with fractions and decimals. As you focus on computation of rational numbers, continue to emphasize addition as combining and addition as joining to a set because students will see these concepts within word problems.

For addition of fractions, we recommend using several models of fractions to help students understand concepts related to addition of fractions. We also recommend demonstrating several algorithms for addition of decimals. Every student should develop efficiency with strategies for addition of fractions and decimals. In the following sections, we provide examples of (1) addition of fractions – like denominators, (2) addition of fractions – unlike denominators, (3) addition of decimals with traditional algorithm, and (4) addition of decimals with partial sums algorithm.

C. Routines and Examples

(1) Addition of Fractions – Like Denominators

Routine

Materials:

- [Module 6 Problem Sets](#)
- [Module 6 Vocabulary Cards](#)
 - If necessary, review Vocabulary Cards before teaching
- A hands-on tool or manipulative like fraction tiles or two-color counters
 - Note that drawings can be used alongside or instead of manipulatives

ROUTINE WITH MANIPULATIVES

Teacher	Let's work on addition. What does it mean to add?
Students	To put together or to join to a set.
Teacher	Addition means to put together or to join to a set. Look at this problem. (Show problem.)
Teacher	First, I see a plus sign (point). The plus sign tells us to add. What does the plus sign mean?
Students	To add.
Teacher	Let's do this problem with fraction tiles. (Move fraction tiles to workspace.)
Teacher	Our first addend is ___. What's our first addend?
Students	___.
Teacher	Let's show this addend by showing the fraction. (Show fraction part compared to whole.)
Teacher	What fraction?

Students ___.

Teacher **Our second addend is ___. What's our second addend?**

Students ___.

Teacher **Let's show the second addend by showing the fraction.**
(Show fraction part compared to whole.)

Teacher **What fraction?**

Students ___.

Teacher **So, we have ___ plus ___. Let's add by combining. What does combining mean?**

Students To put together.

Teacher **Yes. Let's combine, or put together, the parts of the fraction. The parts of the fraction represent the numerator. When adding fractions, first we want to determine whether the denominators are like or unlike. Are the denominators like or the same?**

Students Yes.

Teacher **The denominators are the same. Second, we want to add the parts or numerators of each fraction. That means we have to add ___ one-___ parts and ___ one-___ parts. What do we add?**

Students We add the parts or numerator of the fraction.

Teacher **Let's combine the parts together.**

Students (Combine parts, compare to whole.)

Teacher **So, we now have __, __, __, ... one-___ parts. How many parts?**

Students ___.

Teacher **When you have ___ plus ___, the sum is ___. What's the sum?**

Students ___.

Teacher **___ plus ___ equals ___. Let's say that together.**

Students ___ plus ___ equals ___.

Teacher **So, if you have a set of ___ and a set of ___, when you combine (or put together) the sets, the sum is ___. ___ plus ___ equals ___. Let's review. What's an addend?**

Students One of the sets or numbers added together in an addition problem.

Teacher **What's a sum?**

Students The total number when you combine sets, or the result of adding two or more numbers together.

Teacher **What do you add when you add fractions?**

Students The parts or numerator of each fraction.

Teacher **How could you explain solving this problem to a friend?**

Students We started by showing each addend. Then, we added the parts or numerator together to determine the sum.

ROUTINE WITHOUT MANIPULATIVES

Teacher **Let's work on addition. What does it mean to add?**

Students To put together or to join to a set.

Teacher Addition means to put together or to join to a set. Look at this problem.
(Show problem.)

Teacher First, I see a plus sign (point). The plus sign tells us to add. What does the plus sign mean?

Students To add.

Teacher Our first addend is __. What's our first addend?

Students __.

Teacher Our second addend is __. What's our second addend?

Students __.

Teacher So, we have __ plus __. Let's add by combining. What does combining mean?

Students To put together.

Teacher Yes. Let's combine, or put together, the parts of the fraction. The parts of the fraction are the numerators. When adding fractions, first we want to determine whether the denominators are like or unlike. Are the denominators like or the same?

Students Yes.

Teacher The denominators are the same. The denominator, __, will not change when we add the fractions. Let's go ahead and write the denominator for our sum.
(Write denominator.)

Teacher Now, we want to add the parts or numerator of each fraction. That means we have to add __ one-__ parts and __ one-__ parts. What do we add?

Students We add the parts or numerators of the fraction.

Teacher Let's combine the parts together. What's __ plus __?

Students __.

Teacher Let's write the parts we added together.
(Write parts.)

Teacher When you have __ plus __, the sum is __. What's the sum?

Students __.

Teacher __ plus __ equals __. Let's say that together.

Students __ plus __ equals __.

Teacher So, if you have a set of __ and a set of __, when you combine (or put together) the sets, the sum is __. __ plus __ equals __. Let's review. What's an addend?

Students One of the sets or numbers added together in an addition problem.

Teacher What's a sum?

Students The total number when you combine sets, or the result of adding two or more numbers together.

Teacher What do you add when you add fractions?

Students The parts or numerator of each fraction.

Teacher How could you explain solving this problem to a friend?

Students We determined the denominators of the fraction were the same. We added the parts of the fraction to determine the sum.

Example

$$\frac{2}{8} + \frac{3}{8} = \frac{5}{8}$$

EXAMPLE WITH MANIPULATIVES

- Teacher** Let's work on addition. What does it mean to add?
- Students** To put together or to join to a set.
- Teacher** Addition means to put together or to join to a set. Look at this problem.
(Show problem.)
- Teacher** First, I see a plus sign (point). The plus sign tells us to add. What does the plus sign mean?
- Students** To add.
- Teacher** Let's do this problem with fraction tiles.
(Move fraction tiles to workspace.)
- Teacher** Our first addend is $\frac{2}{8}$. What's our first addend?
- Students** $\frac{2}{8}$.
- Teacher** Let's show this addend by showing the fraction.
(Show 2 one-eighth parts compared to a whole.)
- Teacher** What fraction?
- Students** $\frac{2}{8}$.
- Teacher** Our second addend is $\frac{3}{8}$. What's our second addend?
- Students** $\frac{3}{8}$.
- Teacher** Let's show the second addend by showing the fraction.
(Show 3 one-eighth parts compared to a whole.)
- Teacher** What fraction?
- Students** $\frac{3}{8}$.
- Teacher** So, we have $\frac{2}{8}$ plus $\frac{3}{8}$. Let's add by combining. What does combining mean?
- Students** To put together.
- Teacher** Yes. Let's combine, or put together, the parts of the fraction. The parts of the fractions represent the numerators. When adding fractions, first we want to determine whether the denominators are like or unlike. Are the denominators like or the same?
- Students** Yes.
- Teacher** Both denominators are 8. The denominators are the same or like denominators. Second, we want to add the numerators, or parts, of each fraction. That means we have to add 2 one-eighth parts and 3 one-eighth parts. What do we add?
- Students** We add the parts or numerators of the fraction.
- Teacher** Let's combine the parts together. That means we're combining the numerators.

(Combine parts, compare to whole.)

Teacher So, we now have 1, 2, 3, 4, 5 one-eighth parts. How many parts?

Students 5 one-eighth parts.

Teacher When you have $\frac{2}{8}$ plus $\frac{3}{8}$, the sum is $\frac{5}{8}$. What's the sum?

Students $\frac{5}{8}$.

Teacher $\frac{2}{8}$ plus $\frac{3}{8}$ equals $\frac{5}{8}$. Let's say that together.

Students $\frac{2}{8}$ plus $\frac{3}{8}$ equals $\frac{5}{8}$.

Teacher So, if you have a set of $\frac{2}{8}$ and a set of $\frac{3}{8}$, when you combine (or put together) the parts or numerators of each fraction, the sum is $\frac{5}{8}$. $\frac{2}{8}$ plus $\frac{3}{8}$ equals $\frac{5}{8}$. Let's review. What's an addend?

Students One of the sets or numbers added together in an addition problem.

Teacher What's a sum?

Students The total number when you combine sets, or the result of adding two or more numbers together.

Teacher What do you add when you add fractions?

Students The parts or numerators of each fraction.

Teacher How could you explain solving this problem to a friend?

Students We started by showing each addend. We checked whether there were like denominators, then added the parts or numerators together to determine the sum.

(2) Addition of Fractions – Unlike Denominators

Routine

Materials:

- [Module 6 Problem Sets](#)
- [Module 6 Vocabulary Cards](#)
 - If necessary, review Vocabulary Cards before teaching
- A hands-on tool or manipulative like fraction tiles or two-color counters
 - Note that drawings can be used alongside or instead of manipulatives

ROUTINE WITH MANIPULATIVES

Teacher Let's work on addition. What does it mean to add?

Students To put together or to join to a set.

Teacher Addition means to put together or to join to a set. Look at this problem.

(Show problem.)

Teacher First, I see a plus sign (point). The plus sign tells us to add. What does the plus sign mean?

Students To add.

Teacher Let's do this problem with two-color counters.

(Move two-color counters to workspace.)

Teacher Our first addend is __. What's our first addend?

Students __.

Teacher Let's show this addend by showing the fraction.

(Show set compared to whole with white/yellow counters representing numerator and red counters representing denominator.)

Teacher What fraction?

Students __.

Teacher Our second addend is __. What's our second addend?

Students __.

Teacher Let's show the second addend by showing the fraction.

(Show set compared to whole with white/yellow counters representing numerator and red counters representing denominator.)

Teacher What fraction?

Students __.

Teacher So, we have __ plus __. Let's add by combining. What does combining mean?

Students To put together.

Teacher Yes. Let's combine, or put together, the parts of the fraction. Remember, the parts of the fractions represent the numerators. When adding fractions, first we want to determine whether the denominators are like or unlike. You might also say common or uncommon denominators. Are the denominators the same or alike?

Students No.

Teacher The denominators are not the same. To add, we should add parts or numerators with the same denominator. When the denominators are unlike, the parts or numerators do not have the same value. So, we will work to make the fractions have like denominators. Why do we want to add fractions with like denominators?

Students So, we can add the parts or numerators of the fraction.

Teacher To do this, let's write the first five multiples of each denominator. The first addend has a denominator of __, so let's write the first five multiples of __.

(Write multiples as __, __, __, __, __.)

Teacher What are the multiples of __? Say them with me.

Students __, __, __, __, __.

Teacher The second addend has a denominator of __, so let's write the first five multiples of __.

(Write multiples as __, __, __, __, __.)

Teacher What are the multiples of __? Say them with me.

Students __, __, __, __, __.

Teacher Great. Let's determine the least common multiple of the two fractions. What is the multiple with the least value that you see on both lists of multiples?

Students __.

Teacher So, __ is the least common multiple. Say that with me.

Students Least common multiple.

Teacher Sometimes we call the least common multiple the LCM. What do we call the least common multiple?

Students LCM.

Teacher The least common multiple, or LCM, helps us to determine the common denominator for the two fractions. What does the LCM help with?

Students Finding a common denominator for the two fractions.

Teacher The first addend has a denominator of ____.

OPTION 1: This is the original denominator. We don't have to do anything to this fraction.

OPTION 2: This is not the original denominator. We need to convert the fraction from a denominator of ____ to a denominator of ____.

What do we need to do?

Students **OPTION 1:** We don't have to change the denominator.

OPTION 2: We need to convert the fraction to a denominator of ____.

Teacher **OPTION 2:** To convert the fraction to a denominator of ____, I determine how many groups of ____ (original denominator) I need to make ____ (common denominator). I see I need to make ____, ____, ____ groups of ____ (original denominator). How many groups?

Students ____.

Teacher So, I make ____ groups of ____ with the two-color counters. That means I iterate or copy the original fraction ____ times. What does it mean to iterate?

Students To copy.

Teacher Our new fraction is _____. Is ____ (original fraction) equivalent to ____ (fraction with common denominator)?

Students Yes.

Teacher How do you know the fractions are equivalent?

Students The fractions have the same value. They are equivalent.

Teacher So, we converted the first addend to a common denominator. Let's do the same with the second addend. What's the second addend?

____.

Teacher The second addend has a denominator of ____.

OPTION 1: This is the original denominator. We don't have to do anything to this fraction.

OPTION 2: This is not the original denominator. We need to convert the fraction from a denominator of ____ to a denominator of ____.

What do we need to do?

Students **OPTION 1:** We don't have to change the denominator.

OPTION 2: We need to convert the fraction to a denominator of ____.

Teacher **OPTION 2:** To convert the fraction to a denominator of ____, I determine how many groups of ____ (original denominator) I need to make ____ (common denominator). I see I need to make ____, ____, ____ groups of ____ (original denominator). How many groups?

Students ____.

Teacher We make ___ groups of ___ with the two-color counters. That means I iterate or copy the original fraction ___ times. How many times?

Students ___.

Teacher Let's check our work. Is ___ (original fraction) equivalent to ___ (fraction with common denominator)?

Students Yes.

Teacher How do you know the fractions are equivalent?

Students The fractions have the same value. They are equivalent.

Teacher Now that we have common denominators, we want to add the parts or numerators of each fraction. That means we have to add ___ one-___ parts and ___ one-___ parts. What do we add?

Students We add the parts or numerators of the fraction.

Teacher Let's combine the numerators together. With the two-color counters, we add the red one-___ parts. Because our common denominator is ___, we make groups of ___ (common denominator). We make groups of what?

Students ___.

Teacher We add the one-___ parts. We now have ___, ___, ___, ... one-___ parts. How many parts?

Students ___.

Teacher When you have ___ plus ___, the sum is ___. What's the sum?

Students ___.

Teacher ___ plus ___ equals ___. Let's say that together.

Students ___ plus ___ equals ___.

Teacher So, if you have a set of ___ and a set of ___, when you combine (or put together) the sets, the sum is ___. ___ plus ___ equals ___. Let's review. What's an addend?

Students One of the sets or numbers added together in an addition problem.

Teacher What's a sum?

Students The total number when you combine sets, or the result of adding two or more numbers together.

Teacher What do you add when you add fractions?

Students The parts or numerators of each fraction.

Teacher How could you explain solving this problem to a friend?

Students We started by showing each addend. We decided the denominators were not alike, so we determined a common denominator by using the least common multiples. Then, we added the parts together to determine the sum.

ROUTINE WITHOUT MANIPULATIVES

Teacher Let's work on addition. What does it mean to add?

Students To put together or to join to a set.

Teacher Addition means to put together or to join to a set. Look at this problem.

(Show problem.)

Teacher First, I see a plus sign (point). The plus sign tells us to add. What does the plus sign mean?

Students To add.

Teacher Our first addend is __. What's our first addend?

Students __.

Teacher Our second addend is __. What's our second addend?

Students __.

Teacher So, we have __ plus __. Let's add by combining. What does combining mean?

Students To put together.

Teacher Yes. Let's combine, or put together, the parts of the fraction. Remember, the parts of a fraction represent the numerator. What do you add?

Students The parts or numerators of the fractions.

Teacher When adding fractions, first we want to determine whether the denominators are like or unlike. You might also say common or uncommon denominators. Are the denominators the same or alike?

Students No.

Teacher The denominators are not the same. To add, we should add parts or numerators with the same value. When the denominators are unlike, the parts or numerators do not represent the same value. So, we will work to make the fractions have like denominators. Why do we want to add fractions with like denominators?

Students So, we can add the parts or numerators of the fractions.

Teacher To do this, let's write the first five multiples of each denominator. The first addend has a denominator of __, so let's write the first five multiples of __. (Write multiples as __, __, __, __, __.)

Teacher What are the multiples of __? Say them with me.

Students __, __, __, __, __.

Teacher The second addend has a denominator of __, so let's write the first five multiples of __.

(Write multiples as __, __, __, __, __.)

Teacher What are the multiples of __? Say them with me.

Students __, __, __, __, __.

Teacher Great. Let's determine the least common multiple of the two fractions. What is the multiple with the least value that you see on both lists of multiples?

Students __.

Teacher So, __ is the least common multiple. Say that with me.

Students Least common multiple.

Teacher Sometimes we call the least common multiple the LCM. What do we call the least common multiple?

Students LCM.

Teacher The least common multiple, or LCM, helps us determine the common denominator for the two fractions. What does the LCM help with?

Students Finding a common denominator for the two fractions.



Teacher The first addend has a denominator of ____.

OPTION 1: This is the original denominator. We don't have to do anything to this fraction.

OPTION 2: This is not the original denominator. We need to convert the fraction from a denominator of ____ to a denominator of ____.

What do we need to do?

Students **OPTION 1:** We don't have to change the denominator.

OPTION 2: We need to convert the fraction to a denominator of ____.

Teacher **OPTION 2:** To convert the fraction to a denominator of ____, I determine how many groups of ____ (original denominator) I need to make ____ (common denominator). I see I need to make ____, ____, ____ groups of ____ (original denominator). **How many groups?**

Students ____.

Teacher **So, I multiply the denominator times ____ and the numerator times ____.** Let's multiply the denominator first. ____ (original denominator) times ____ is what?

Students ____.

Teacher **That's right. ____ times ____ equals ____.** Our new denominator is ____.

What's our new denominator?

Students ____.

Teacher **Now, let's multiply the numerator times ____.** ____ (original numerator) times ____ is what?

Students ____.

Teacher **Yes. ____ times ____ equals ____.** Our new numerator is ____.

What's the new numerator?

Students ____.

Teacher **Let's check our work. Is ____ (original fraction) equivalent to ____ (fraction with common denominator)?** **How do you know the fractions are equivalent?**

Students The fractions have the same value. They are equivalent.

Teacher **So, we converted the first addend to a common denominator. Let's do the same with the second addend. What's the second addend?**

____.

Teacher The second addend has a denominator of ____.

OPTION 1: This is the original denominator. We don't have to do anything to this fraction.

OPTION 2: This is not the original denominator. We need to convert the fraction from a denominator of ____ to a denominator of ____.

What do we need to do?

Students **OPTION 1:** We don't have to change the denominator.

OPTION 2: We need to convert the fraction to a denominator of ____.

Teacher **OPTION 2:** To convert the fraction to a denominator of ____, I determine how many groups of ____ (original denominator) I need to make ____

(common denominator). I see I need to make __, __, __ groups of __ (original denominator). **How many groups?**

Students
Teacher
____.
So, I multiply the denominator times __ and the numerator times __. Let's multiply the denominator first. __ (original denominator) times __ is what?

Students
Teacher
____.
That's right. __ times __ equals __. Our new denominator is __. What's our new denominator?

Students
Teacher
____.
Now, let's multiply the numerator times __. __ (original numerator) times __ is what?

Students
Teacher
____.
Yes. __ times __ equals __. Our new numerator is __. What's the new numerator?

Students
Teacher
____.
Let's check our work. Is __ (original fraction) equivalent to __ (fraction with common denominator)?

Students
Teacher
Yes.
How do you know the fractions are equivalent?

Students
Teacher
The fractions have the same value. They are equivalent.

Now that we have common denominators, we want to add the parts or numerator of each fraction. That means we have to add __ one-__ parts and __ one-__ parts. What do we add?

Students
Teacher
We add the parts of the fraction.
Let's combine the parts or numerators together.
(Combine parts, compare to whole.)

Teacher
Students
So, we now have __, __, __, ... one-__ parts. How many parts?

Teacher
Students
____.
When you have __ plus __, the sum is __. What's the sum?

Teacher
Students
____.
__ plus __ equals __. Let's say that together.

Teacher
Students
____ plus __ equals ____.

Teacher
So, if you have a set of __ and a set of __, when you combine (or put together) the sets, the sum is __. __ plus __ equals __. Let's review. What's an addend?

Students
Teacher
One of the sets or numbers added together in an addition problem.
What's a sum?

Students
The total number when you combine sets, or the result of adding two or more numbers together.

Teacher
What do you add when you add fractions?

Students
The parts or numerator of each fraction.

Teacher
How could you explain solving this problem to a friend?

Students We started by showing each addend. We used least common multiples to help determine common denominators. Then, we added the parts together to determine the sum.

Example

$$\frac{3}{4} + \frac{1}{3} = \frac{13}{12}$$

EXAMPLE WITH MANIPULATIVES

Teacher Let's work on addition. What does it mean to add?

Students To put together or to join to a set.

Teacher Addition means to put together or to join to a set. Look at this problem. (Show problem.)

Teacher First, I see a plus sign (point). The plus sign tells us to add. What does the plus sign mean?

Students To add.

Teacher Let's do this problem with two-color counters. (Move two-color counters to workspace.)

Teacher Our first addend is $\frac{3}{4}$. What's our first addend?

Students $\frac{3}{4}$.

Teacher Let's show this addend by showing the fraction. First, we have a denominator of 4, so let's show 4 yellow counters. How many?

Students 4.

Teacher Then, we need to show 3 of the 4 parts as red to show $\frac{3}{4}$. How many should we make red?

Students 3.

Teacher What fraction?

Students $\frac{3}{4}$.

Teacher Our second addend is $\frac{1}{3}$. What's our second addend?

Students $\frac{1}{3}$.

Teacher Let's show the second addend by showing the fraction. First, we have a denominator of 3, so let's show 3 yellow counters. How many?

Students 3.

Teacher Then, we need to show 1 of the 3 parts as red to show $\frac{1}{3}$. How many should we make red?

Students 1.

Teacher What fraction?

Students $\frac{1}{3}$.

Teacher So, we have $\frac{3}{4}$ plus $\frac{1}{3}$. Let's add by combining. What does combining mean?

Students To put together.

Teacher Yes. Let's combine, or put together, the parts of the fraction. When adding fractions, first we want to determine whether the denominators are like or unlike. You might also say common or uncommon denominators. Are the denominators the same or alike?

Students No.

Teacher How do you know the denominators are not alike?

Students We have a denominator of 4 and a denominator of 3. Those are not the same.

Teacher The denominators are not the same. To add, we should add parts or numerators with the same denominator. When the denominators are unlike, the parts or numerators do not represent the same value. So, we will work to make the fractions have like denominators. Why do we want to add fractions with like denominators?

Students So, we can add the parts or numerator of the fraction.

Teacher To do this, let's write the first five multiples of each denominator. The first addend has a denominator of 4, so let's write the first five multiples of 4. (Write multiples as 4, 8, 12, 16, 20.)

Teacher What are the multiples of 4? Say them with me.

Students 4, 8, 12, 16, 20.

Teacher The second addend has a denominator of 3, so let's write the first five multiples of 3. (Write multiples as 3, 6, 9, 12, 15.)

Teacher What are the multiples of 3? Say them with me.

Students 3, 6, 9, 12, 15.

Teacher Great. Let's determine the least common multiple of the two fractions. What is the multiple with the least value that you see on both lists of multiples?

Students 12.

Teacher So, 12 is the least common multiple. What is 12?

Students The least common multiple.

Teacher Sometimes we call the least common multiple the LCM. What do we call the least common multiple?

Students LCM.

Teacher The least common multiple, or LCM, helps us determine the common denominator for the two fractions. What does the LCM help with?

Students Finding a common denominator for the two fractions.

Teacher The first addend has a denominator of 4, which is not the original denominator. We need to convert the fraction from a denominator of 4 to a denominator of 12. What do we need to do?

Students Convert the fraction from a denominator of 4 to a denominator of 12.

Teacher To convert the fraction to a denominator of 12, I determine how many groups of 4 I need to make 12. I see I need to make 1, 2, 3 groups of 4. (Point to the multiples of 4, 8, and 12.) How many groups?

Students 3.

Teacher **Let's make 3 groups of the fraction $\frac{3}{4}$ with the two-color counters. We already have one group of $\frac{3}{4}$. Let's make a second group (show 3 red counters and 1 yellow counter) and a third group (show 3 red counters and 1 yellow counter.) Our new fraction is $\frac{9}{12}$. Is $\frac{9}{12}$ equivalent to $\frac{3}{4}$?**

Students Yes. The fractions are equivalent.

Teacher **So, we converted the first addend to a common denominator. Let's do the same with the second addend. What's the second addend?**

Students $\frac{1}{3}$.

Teacher **The second addend has a denominator of 3, which is not the original denominator. We need to convert the fraction from a denominator of 3 to a denominator of 12. What do we need to do?**

Students Convert the fraction from a denominator of 3 to a denominator of 12.

Teacher **To convert the fraction to a denominator of 12, I determine how many groups of 3 I need to make 12. I see I need to make 1, 2, 3, 4 groups of 3. (Point to the multiples of 3, 6, 9, and 12.) How many groups?**

Students 4.

Teacher **Let's make 4 groups of the fraction $\frac{1}{3}$ with the two-color counters. We already have one group of $\frac{1}{3}$. Let's make a second group (show 1 red counter and 2 yellow counters), a third group (show 1 red counter and 2 yellow counters), and a fourth group (show 1 red counter and 2 yellow counters). Our new fraction is $\frac{4}{12}$. Is $\frac{4}{12}$ equivalent to $\frac{1}{3}$?**

Students Yes. The fractions are equivalent.

Teacher **Now that we have common denominators, we want to add the parts or numerators of each fraction. That means we have to add 9 one-twelfth parts and 4 one-twelfth parts. What do we add?**

Students We add the parts or numerators of the fractions.

Teacher **Let's combine the parts or numerators together. With the two-color counters, we add the red one-twelfth parts. Because our common denominator is 12, we make groups of 12 (common denominator). We make groups of what?**

Students 12.

Teacher **We add the one-twelfth parts. We now have 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13 one-twelfth parts. How many parts?**

Students 13.

Teacher **When you have $\frac{9}{12}$ plus $\frac{4}{12}$, the sum is $\frac{13}{12}$. What's the sum?**

Students $\frac{13}{12}$.

Teacher **$\frac{9}{12}$ plus $\frac{4}{12}$ equals $\frac{13}{12}$. Let's say that together.**

Students $\frac{9}{12}$ plus $\frac{4}{12}$ equals $\frac{13}{12}$.

$\frac{13}{12}$ is also equivalent to $1\frac{1}{12}$.

Teacher If you have a set of $\frac{3}{4}$ and a set of $\frac{1}{3}$, when you combine (or put together) the sets, the sum is $\frac{13}{12}$. $\frac{9}{12}$ plus $\frac{4}{12}$ equals $\frac{13}{12}$. Let's review. What's an addend?

Students One of the sets or numbers added together in an addition problem.

Teacher What's a sum?

Students The total number when you combine sets, or the result of adding two or more numbers together.

Teacher What do you add when you add fractions?

Students The parts or numerator of each fraction.

Teacher How could you explain solving this problem to a friend?

Students We started by showing each addend. We determined the denominators were not alike. So, we used least common multiples to find a common denominator. After converting both fractions to a common denominator, we added the parts or numerators together to determine the sum.

(3) Addition of Decimals with Traditional Algorithm

Routine

Materials:

- [Module 6 Problem Sets](#)
- [Module 6 Vocabulary Cards](#)
 - If necessary, review Vocabulary Cards before teaching
- A hands-on tool or manipulative like Base-10 blocks or money
 - Note that drawings can be used alongside or instead of manipulatives

ROUTINE WITH MANIPULATIVES

Teacher Let's work on addition. What does it mean to add?

Students To put together or to join to a set.

Teacher Addition means to put together or to join to a set. Look at this problem. (Show problem.)

Teacher First, I see a plus sign (point). The plus sign tells us to add. What does the plus sign mean?

Students To add.

Teacher Let's do this problem with Base-10 blocks. (Move Base-10 blocks to workspace.)

Teacher When we use the Base-10 blocks with decimals, we can shift the meaning of each type of block. Today, let's use the flats to represent ones. What do the flats represent?

Students Ones.

Teacher We'll use the rods to represent tenths. What do the rods represent?

Students Tenths.

Teacher **How can we use the rods to represent tenths?**

Students 1 rod equals 1 tenth.

Teacher **What do you notice about the relationship between the rods and the flat?**

Students There are 10 tenths in 1 in the same way there are 10 rods in 1 flat.

Teacher **With our Base-10 blocks, the units represent hundredths. What do the units represent?**

Students Hundredths.

Teacher **What do you notice about the relationship between the units and the rods?**

Students There are 10 hundredths in 1 tenth in the same way there are 10 units in 1 rod.

Teacher **Our first addend is __. What's our first addend?**

Students __.

Teacher **Let's show this addend by showing __ ones, __ tenths, and __ hundredths. (Show with Base-10 blocks.)**

Teacher **How many?**

Students __.

Teacher **Our second addend is __. What's our second addend?**

Students __.

Teacher **Let's show the second addend by showing __ ones, __ tenths, and __ hundredths. (Show with Base-10 blocks. Place Base-10 blocks under the first addend.)**

Teacher **How many?**

Students __.

Teacher **So, we have __ plus __. Let's add by combining. What does combining mean?**

Students To put together.

Teacher **Yes. Let's combine or put together. First, let's combine the least place value. That means the place value with the least or smallest value. What's the least place value in this problem?**

Students Hundredths.

Teacher **Let's add the hundredths together. (Move two sets of hundredths together.)**

Teacher **Let's count to learn the sum of the hundredths. (Count hundredths.)**

Teacher **How many hundredths are there in total or altogether?**

Students __.

Teacher **Yes! There are __ hundredths. If we have more than 9 hundredths, we have to regroup. Do we have more than 9 hundredths?**

Students Yes.

Teacher **We have more than 9 hundredths. That means we have to regroup. To regroup, we count 10 hundredths and regroup/trade/exchange the 10 hundredths for 1 tenth. Let's do that together. Let's count out 10 hundredths. (Count 10 hundredths.)**

Teacher **Let's regroup/trade/exchange the 10 hundredths for 1 tenth. See how 1 tenth is the same as 10 hundredths?**

Students Yes.

Teacher **We leave the remaining hundredths here. But we can't put this 1 tenth in the hundredths place. The hundredths place is only for hundredths. So, we place the 1 tenth in the tenths column. I like to place the 1 tenth above the other tenths.**
(Place 1 tenth above tenths column.)

Teacher **Now, let's combine the tenths. That means we put all the tenths together.**
(Move sets of tenths together.)

Teacher **How many tenths are there in total or altogether?**

Students ___.

Teacher **There are ___ tenths. If we have more than 9 tenths, we have to regroup. Do we have more than 9 tenths?**

Students No.

Teacher **Now, let's combine the ones. Let's put all the ones together.**
(Move sets of ones together.)

Teacher **How many ones are there in total or altogether?**

Students ___.

Teacher **So, let's count the ones, tenths, and hundredths to learn the sum. Ready?**
(Count the ones, then tenths, then hundredths.)

Teacher **That means ___ plus ___ equals ___. Let's say that together.**

Students ___ plus ___ equals ___.

Teacher **Let's say it together again.**

Students ___ plus ___ equals ___.

Teacher **So, if you have a set of ___ and a set of ___, when you combine (or put together) the sets, the sum is ___. ___ plus ___ equals ___. Let's review. What's an addend?**

Students One of the sets or numbers added together in an addition problem.

Teacher **What's a sum?**

Students The total number when you combine sets, or the result of adding two or more numbers together.

Teacher **What does it mean to regroup/trade/exchange?**

Students You can regroup/trade/exchange 10 hundredths for 1 tenth.

Teacher **How could you explain solving this problem to a friend?**

Students We started by showing each addend. Then, we combined the hundredths. We regrouped 10 hundredths for 1 tenth. Then, we combined the tenths. Then, we combined the ones. The sum was the total number of ones, tenths, and hundredths.

ROUTINE WITHOUT MANIPULATIVES

Teacher **Let's work on addition. What does it mean to add?**

Students To put together or to join to a set.

Teacher **Addition means to put together or to join to a set. Look at this problem.**
(Show problem.)

Teacher First, I see a plus sign (point). The plus sign tells us to add. What does the plus sign mean?

Students To add.

Teacher Let's do this problem with our pencil. First, when I see a problem like this that requires computation, I like to draw vertical lines to separate the different place value columns. Let's draw a vertical line between the ones column and the tenths column and another line between the tenths column and the hundredths column.
(Draw vertical lines to separate place value columns.)

Teacher Now, we start by adding the hundredths. What should we add first?

Students The hundredths.

Teacher Which hundredths do we add?

Students ___ plus ___.

Teacher What's ___ plus ___?
(If a student has difficulty with addition, say: Start with the greater addend. Place that number in your fist, and let's count up ___ more. Ready? __: __, __, __. See Counting Up poster at the end of Module 4 for more information.)

Teacher How many hundredths are there in total or altogether?

Students ___.

Teacher Yes! There are ___ hundredths. If we have more than 9 hundredths, we have to regroup. Do we have more than 9 hundredths?

Students Yes.

Teacher We have more than 9 hundredths. That means we have to regroup. We think of our hundredths sum as 1 tenth and ___ hundredths. We write the hundredths in the hundredths column under the equal line.
(Write hundredths under equal line.)

Teacher We regroup the 1 tenth to the tenths column. We write the 1 tenth in the tenths column above the other tenths.
(Write 1 above tenths column.)

Teacher Now, let's add the tenths. Which tens do we add?

Students ___ plus ___ plus ___.

Teacher What's ___ plus ___ plus ___?

Students ___.

Teacher How many tenths are there in total or altogether?

Students ___.

Teacher There are ___ tenths. If we have more than 9 tenths, we have to regroup. Do we have more than 9 tenths?

Students No.

Teacher Now, let's add the ones. Which ones do we add?

Students ___ plus ___.

Teacher What's ___ plus ___?

Students ___.

Teacher How many ones are there in total or altogether?

Students ___.

Teacher So, let's look at the problem. What's ___ plus ___?
Students ___.
Teacher That's right. ___ plus ___ equals ___. Let's say that together.
Students ___ plus ___ equals ___.
Teacher So, if you have a set of ___ and a set of ___, when you combine (or join) the sets, the sum is ___. ___ plus ___ equals ___. Let's review. What's an addend?
Students One of the sets or numbers added together in an addition problem.
Teacher What's a sum?
Students The total number when you combine sets, or the result of adding two or more numbers together.
Teacher What does it mean to regroup/trade/exchange?
Students You can regroup/trade/exchange 10 hundredths for 1 tenth.
Teacher How could you explain solving this problem to a friend?
Students First, we combined the hundredths. We regrouped 10 hundredths for 1 tenth. Then, we combined the tenths. Then, we combined the ones. The sum was the total number of ones, tenths, and hundredths.

Example

2.16
+ 4.78

6.94

EXAMPLE WITH MANIPULATIVES

Teacher Let's work on addition. What does it mean to add?
Students To put together or to join to a set.
Teacher Addition means to put together or to join to a set. Look at this problem.
 (Show problem.)
Teacher First, I see a plus sign (point). The plus sign tells us to add. What does the plus sign mean?
Students To add.
Teacher Let's do this problem with Base-10 blocks.
 (Move Base-10 blocks to workspace.)
Teacher When we use the Base-10 blocks with decimals, we can shift the meaning of each type of block. Today, let's use the flats to represent ones. What do the flats represent?
Students Ones.
Teacher We'll use the rods to represent tenths. What do the rods represent?
Students Tenths.
Teacher How can we use the rods to represent tenths? What do you notice about the relationship between the rods and the flat?
Students There are 10 tenths in 1 in the same way there are 10 rods in 1 flat.

Teacher With our Base-10 blocks, the units represent hundredths. What do the units represent?

Students Hundredths.

Teacher What do you notice about the relationship between the units and the rods?

Students There are 10 hundredths in 1 tenth in the same way there are 10 units in 1 rod.

Teacher Our first addend is 2 and 16 hundredths. What's our first addend?

Students 2 and 16 hundredths.

Teacher Let's show this addend by showing 2 ones, 1 tenth, and 6 hundredths. (Show with Base-10 blocks.)

Teacher How many?

Students 2 and 16 hundredths.

Teacher Our second addend is 4 and 78 hundredths. What's our second addend?

Students 4 and 78 hundredths.

Teacher Let's show the second addend by showing 4 ones, 7 tenths, and 8 hundredths. (Show with Base-10 blocks. Place Base-10 blocks under the first addend.)

Teacher How many?

Students 4 and 78 hundredths.

Teacher So, we have 2 and 16 hundredths plus 4 and 78 hundredths. Let's add by combining. What does combining mean?

Students To put together.

Teacher Yes. Let's combine or put together. First, let's combine the least place value. What's the least place value in this problem?

Students Hundredths.

Teacher Let's add the hundredths together. 6 hundredths plus 8 hundredths. (Move two sets of hundredths together.)

Teacher Let's count to learn the sum of the hundredths. (Count hundredths.)

Teacher How many hundredths are there in total or altogether?

Students 14.

Teacher Yes! There are 14 hundredths. If we have more than 9 hundredths, we have to regroup. Do we have more than 9 hundredths?

Students Yes.

Teacher We have more than 9 hundredths. That means we have to regroup. To regroup, we count 10 hundredths and regroup/trade/exchange the 10 hundredths for 1 tenth. Let's do that together. Let's count out 10 hundredths. (Count 10 hundredths.)

Teacher Let's regroup/trade/exchange the 10 hundredths for 1 tenth. See how 1 tenth is the same as 10 hundredths?

Students Yes.

Teacher We leave the remaining hundredths here. But we can't put this 1 tenth in the hundredths place. The hundredths place is only for hundredths. So, we place the 1 tenth in the tenths column. I like to place the 1 tenth above the other tenths. (Place 1 tenth above tenths column.)

Teacher Now, let's combine the tenths. That means we put all the tenths together.
(Move sets of tenths together.)

Teacher Let's add 1 tenth plus 1 tenth plus 7 tenths. How many tenths are there in total or altogether?

Students 9.

Teacher There are 9 tenths. If we have more than 9 tenths, we have to regroup. Do we have more than 9 tenths?

Students No.

Teacher Now, let's combine the ones. Let's put all the ones together.
(Move sets of ones together.)

Teacher How many ones are there in total or altogether?

Students 6.

Teacher So, let's count the ones, tenths, and hundredths to learn the sum. Ready?
(Count the ones, then tenths, then hundredths.)

Teacher That means 2 and 16 hundredths plus 4 and 78 hundredths equals 6 and 94 hundredths. Let's say that together.

Students 2 and 16 hundredths plus 4 and 78 hundredths equals 6 and 94 hundredths.

Teacher Let's say it together again.

Students 2 and 16 hundredths plus 4 and 78 hundredths equals 6 and 94 hundredths.

Teacher Let's review. What's an addend?

Students One of the sets or numbers added together in an addition problem.

Teacher What's a sum?

Students The total number when you combine sets, or the result of adding two or more numbers together.

Teacher What does it mean to regroup/trade/exchange?

Students You can regroup/trade/exchange 10 hundredths for 1 tenth.

Teacher How could you explain solving this problem to a friend?

Students We started by showing each addend. Then, we combined the hundredths. We regrouped 10 hundredths for 1 tenth. Then, we combined the tenths. Then, we combined the ones. The sum was the total number of ones, tenths, and hundredths.

(4) Addition of Decimals with Partial Sums Algorithm

Routine

Materials:

- [Module 6 Problem Sets](#)
- [Module 6 Vocabulary Cards](#)
 - If necessary, review Vocabulary Cards before teaching
- A hands-on tool or manipulative like Base-10 blocks or money
 - Note that drawings can be used alongside or instead of manipulatives

ROUTINE WITH MANIPULATIVES

- Teacher** Let's work on addition. What does it mean to add?
Students To put together or to join to a set.
- Teacher** Addition means to put together or to join to a set. Look at this problem.
(Show problem.)
- Teacher** First, I see a plus sign (point). The plus sign tells us to add. What does the plus sign mean?
Students To add.
- Teacher** Let's do this problem with money.
(Move money to workspace.)
- Teacher** When we use the money, the dollar bills represent ones. What do the dollar bills represent?
Students Ones.
- Teacher** We'll use the dimes to represent tenths. What do the dimes represent?
Students Tenths.
- Teacher** How can we use the dimes to represent tenths?
Students 1 dime represents 1 tenth.
- Teacher** What do you notice about the relationship between the dimes and the dollar bill?
Students There are 10 dimes in 1 dollar.
- Teacher** With our money, the pennies represent hundredths. What do the pennies represent?
Students Hundredths.
- Teacher** What do you notice about the relationship between the pennies and the dimes?
Students There are 10 pennies in 1 dime.
- Teacher** Our first addend is __. What's our first addend?
Students __.
- Teacher** Let's show this addend by showing __ ones, __ tenths, and __ hundredths.
(Show with money.)
- Teacher** How many?
Students __.
- Teacher** Our second addend is __. What's our second addend?
Students __.
- Teacher** Let's show the second addend by showing __ ones, __ tenths, and __ hundredths.
(Show with money. Place under the first addend.)
- Teacher** How many?
Students __.
- Teacher** So, we have __ plus __. Let's add by combining. What does combining mean?
Students To put together.

Teacher Yes. Let's combine or put together. First, let's combine the ones. That means we combine the dollars. This will be our first partial sum. It's the sum for part of the problem. Adding the ones means we put all the ones together. (Move two sets of ones together.)

Teacher Let's count to learn the sum of the ones. (Count ones.)

Teacher How many ones are there in total or altogether?
Students __.

Teacher This __ is one of our partial sums. It's the sum of the ones. Now, let's combine the tenths. That means we put all the dimes together. (Move dimes together.)

Teacher How many dimes are there in total or altogether?
Students __.

Teacher This __ is another of our partial sums. It's the sum of the tenths. What's a partial sum?
Students It's a sum of part of the problem.

Teacher Let's combine the hundredths or pennies. Let's put all the hundredths together to get the sum of the hundredths. (Move pennies together.)

Teacher How many pennies are there in total or altogether?
Students __.

Teacher Now, we add the partial sums. Let's add the partial sums of the ones, tenths, and hundredths or the dollars, dimes, and pennies. (Start with dollars, then add the dimes, then add the pennies.)

Teacher That means __ plus __ equals __. Let's say that together.
Students __ plus __ equals __.

Teacher Let's say it together again.
Students __ plus __ equals __.

Teacher So, if you have a set of __ and a set of __, when you combine (or put together) the sets, the sum is __. __ plus __ equals __. Let's review. What's an addend?
Students One of the sets or numbers added together in an addition problem.

Teacher What's a sum?
Students The total number when you combine sets, or the result of adding two or more numbers together.

Teacher How could you explain solving this problem to a friend?
Students We started by showing each addend. Then, we combined the ones. Then, we combined the tenths. Then, we combined the hundredths. We added the partial sums of the ones, tenths, and hundredths by adding the dollars, dimes, and pennies. The sum was the total number of ones, tenths, and hundredths.

ROUTINE WITHOUT MANIPULATIVES

- Teacher** Let's work on addition. What does it mean to add?
Students To put together or to join to a set.
- Teacher** Addition means to put together or to join to a set. Look at this problem.
(Show problem.)
- Teacher** First, I see a plus sign (point). The plus sign tells us to add. What does the plus sign mean?
Students To add.
- Teacher** Let's do this problem with our pencil. First, when I see a problem like this that requires computation, I like to draw vertical lines to separate the different place value columns. Let's draw a vertical line between the ones column and the tenths column and another line between the tenths column and the hundredths column.
(Draw vertical lines to separate place value columns.)
- Teacher** With the partial sums algorithm, we start by adding the greatest place value. What should we add first?
Students The ones.
- Teacher** Which ones do we add?
Students ___ plus ___.
- Teacher** What's ___ plus ___?
(If a student has difficulty with addition, say: **Start with the greater addend. Place that number in your fist, and let's count up ___ more. Ready? ___: __, __, __.** See Counting Up poster at the end of Module 4 for more information.)
- Teacher** How many ones are there in total or altogether?
Students ___.
- Teacher** So, let's write ___ under the equal line.
(Write ones.)
- Teacher** Now, let's add the tenths. Which tens do we add?
Students ___ plus ___.
- Teacher** What's ___ plus ___?
Students ___.
- Teacher** Let's write ___ under the equal line.
(Write tenths.)
- Teacher** Now, let's add the hundredths. Which hundredths do we add?
Students ___ plus ___.
- Teacher** What's ___ plus ___?
Students ___.
- Teacher** Let's write ___ under the equal line.
(Write hundredths.)
- Teacher** Now, let's add the partial sums. What's ___ plus ___ plus ___?
Students ___.
- Teacher** That's right. To review, ___ plus ___ equals __. Let's say that together.
Students ___ plus ___ equals ___.

Teacher So, if you have a set of __ and a set of __, when you combine (or join) the sets, the sum is __. __ plus __ equals __. Let's review. What's an addend?

Students One of the sets or numbers added together in an addition problem.

Teacher What's a sum?

Students The total number when you combine sets, or the result of adding two or more numbers together.

Teacher What's a partial sum?

Students The sum of just the ones or the tenths or the hundredths.

Teacher How could you explain solving this problem to a friend?

Students First, we combined the ones. Then, we combined the tenths. Then, we combined the hundredths. The sum was the total number of ones, tenths, and hundredths.

2.16
+ 4.78
6.94

Example

EXAMPLE WITH MANIPULATIVES

Teacher Let's work on addition. What does it mean to add?

Students To put together or to join to a set.

Teacher Addition means to put together or to join to a set. Look at this problem. (Show problem.)

Teacher First, I see a plus sign (point). The plus sign tells us to add. What does the plus sign mean?

Students To add.

Teacher Let's do this problem with Base-10 blocks. (Move Base-10 blocks to workspace.)

Teacher When we use the Base-10 blocks with decimals, we can shift the meaning of each type of block. Today, let's use the flats to represent ones. What do the flats represent?

Students Ones.

Teacher We'll use the rods to represent tenths. What do the rods represent?

Students Tenths.

Teacher How can we use the rods to represent tenths?

Students 1 rod equals 1 tenth.

Teacher What do you notice about the relationship between the rods and the flat?

Students There are 10 tenths in 1 in the same way there are 10 rods in 1 flat.

Teacher With our Base-10 blocks, the units represent hundredths. What do the units represent?

Students Hundredths.

Teacher What do you notice about the relationship between the units and the rods?

Students There are 10 hundredths in 1 tenth in the same way there are 10 units in 1 rod.

Teacher Our first addend is 2 and 16 hundredths. What's our first addend?

Students 2 and 16 hundredths.

Teacher **Let's show this addend by showing 2 ones, 1 tenth, and 6 hundredths.**
(Show with Base-10 blocks.)

Teacher **How many?**
Students 2 and 16 hundredths.

Teacher **Our second addend is 4 and 78 hundredths. What's our second addend?**
Students 4 and 78 hundredths.

Teacher **Let's show the second addend by showing 4 ones, 7 tenths, and 8 hundredths.**
(Show with Base-10 blocks. Place Base-10 blocks under the first addend.)

Teacher **How many?**
Students 4 and 78 hundredths.

Teacher **So, we have 2 and 16 hundredths plus 4 and 78 hundredths. Let's add by combining. What does combining mean?**
Students To put together.

Teacher **Yes. Let's combine or put together. We'll use the partial sums strategy. What strategy?**
Students Partial sums.

Teacher **With the partial sums strategy, we add the greatest place value first. What's the greatest place value in this problem?**
Students Ones.

Teacher **Let's add the ones together: 2 plus 4.**
(Move 2 flats and 4 flats together.)

Teacher **Let's count to learn the sum of the ones.**
(Count ones.)

Teacher **How many ones are there in total or altogether?**
Students 6.

Teacher **Yes! There are 6 ones. Now, let's combine the tenths. That means we put all the tenths together: 1 tenth and 7 tenths.**
(Move 1 rod and 7 rods together.)

Teacher **How many tenths are there in total or altogether?**
Students 8.

Teacher **There are 8 tenths. Now, let's combine the hundredths. Let's put all the hundredths together: 6 hundredths and 8 hundredths.**
(Move 6 units and 8 units together.)

Teacher **How many hundredths are there in total or altogether?**
Students 14.

Teacher **Notice that 14 hundredths is the same as what?**
Students 1 tenth and 4 hundredths.

Teacher **So, let's count the ones, tenths, and hundredths to learn the sum. Ready?**
6 and 10, 20, 30, 40, 50, 60, 70, 80, 90, 91, 92, 93, 94 hundredths.

Teacher **That means 2 and 16 hundredths plus 4 and 78 hundredths equals 6 and 94 hundredths. Let's say that together.**
Students 2 and 16 hundredths plus 4 and 78 hundredths equals 6 and 94 hundredths.

Teacher **Let's say it together again.**
Students 2 and 16 hundredths plus 4 and 78 hundredths equals 6 and 94 hundredths.

Teacher **Let's review. What's an addend?**
Students One of the sets or numbers added together in an addition problem.

Teacher **What's a sum?**
Students The total number when you combine sets, or the result of adding two or more numbers together.

Teacher **What's a partial sum?**
Students The sum of just the ones or the tenths or the hundredths.

Teacher **How could you explain solving this problem to a friend?**
Students We started by showing each addend. Then, we added the ones, then the tenths, and then the hundredths. The sum was the total number of ones, tenths, and hundredths.

D. Problems for Use During Instruction

[See Module 6 Problem Sets.](#)

E. Vocabulary Cards for Use During Instruction

[See Module 6 Vocabulary Cards.](#)

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Module 6: Addition of Rational Numbers

Problem Sets

- A. Proper fractions with like denominators and sums <1 (20)
- B. Improper fractions with like denominators and sums >1 (10)
- C. Mixed numbers with like denominators and sums >1 (10)
- D. Proper fractions with unlike denominators and sums <1 (20)
- E. Improper fractions with unlike denominators and sums >1 (10)
- F. Mixed numbers with unlike denominators and sums >1 (10)

- G. Decimals with tenths; no regrouping (20)
- H. Decimals with tenths; regrouping (20)
- I. Decimals with hundredths; no regrouping (20)
- J. Decimals with hundredths; regrouping (20)
- K. Decimals with tenths and hundredths; mix of regrouping (20)

A.

$$\frac{2}{5} + \frac{2}{5} =$$

A.

$$\frac{4}{10} + \frac{3}{10} =$$

A.

$$\frac{3}{6} + \frac{1}{6} =$$

A.

$$\frac{2}{4} + \frac{1}{4} =$$

A.

$$\frac{1}{3} + \frac{1}{3} =$$

A.

$$\frac{2}{6} + \frac{3}{6} =$$

A.

$$\frac{3}{8} + \frac{4}{8} =$$

A.

$$\frac{4}{10} + \frac{1}{10} =$$

A.

$$\frac{2}{12} + \frac{4}{12} =$$

A.

$$\frac{7}{12} + \frac{3}{12} =$$

A.

$$\frac{5}{9} + \frac{2}{9} =$$

A.

$$\frac{3}{5} + \frac{1}{5} =$$

A.

$$\frac{2}{6} + \frac{1}{6} =$$

A.

$$\frac{4}{7} + \frac{1}{7} =$$

A.

$$\frac{3}{9} + \frac{4}{9} =$$

A.

$$\frac{5}{10} + \frac{2}{10} =$$

A.

$$\frac{1}{4} + \frac{1}{4} =$$

A.

$$\frac{1}{6} + \frac{1}{6} =$$

A.

$$\frac{2}{7} + \frac{3}{7} =$$

A.

$$\frac{1}{8} + \frac{2}{8} =$$

B.

$$\frac{6}{5} + \frac{7}{5} =$$

B.

$$\frac{12}{8} + \frac{3}{8} =$$

B.

$$\frac{7}{6} + \frac{3}{6} =$$

B.

$$\frac{5}{4} + \frac{1}{4} =$$

B.

$$\frac{2}{3} + \frac{4}{3} =$$

B.

$$\frac{8}{6} + \frac{3}{6} =$$

B.

$$\frac{5}{8} + \frac{9}{8} =$$

B.

$$\frac{11}{10} + \frac{13}{10} =$$

B.

$$\frac{13}{12} + \frac{4}{12} =$$

B.

$$\frac{10}{10} + \frac{5}{10} =$$

c.

$$7\frac{7}{12} + 4\frac{3}{12} =$$

c.

$$\frac{3}{5} + 2\frac{3}{5} =$$

c.

$$1\frac{2}{6} + 3\frac{5}{6} =$$

c.

$$\frac{8}{5} + 4\frac{1}{5} =$$

c.

$$1\frac{4}{9} + 2\frac{6}{9} =$$

c.

$$1\frac{10}{12} + 3\frac{5}{12} =$$

c.

$$1\frac{3}{4} + 1\frac{3}{4} =$$

c.

$$7\frac{5}{6} + 2\frac{7}{6} =$$

c.

$$\frac{3}{4} + 2\frac{3}{4} =$$

c.

$$1\frac{6}{8} + 3\frac{5}{8} =$$

D.

$$\frac{2}{4} + \frac{1}{3} =$$

D.

$$\frac{1}{2} + \frac{2}{6} =$$

D.

$$\frac{2}{12} + \frac{1}{4} =$$

D.

$$\frac{3}{10} + \frac{1}{5} =$$

D.

$$\frac{1}{6} + \frac{1}{3} =$$

D.

$$\frac{2}{10} + \frac{2}{5} =$$

D.

$$\frac{3}{8} + \frac{2}{4} =$$

D.

$$\frac{3}{6} + \frac{1}{3} =$$

D.

$$\frac{1}{3} + \frac{1}{2} =$$

D.

$$\frac{2}{4} + \frac{3}{8} =$$

D.

$$\frac{3}{10} + \frac{2}{5} =$$

D.

$$\frac{2}{12} + \frac{5}{6} =$$

D.

$$\frac{1}{3} + \frac{2}{5} =$$

D.

$$\frac{1}{3} + \frac{1}{4} =$$

D.

$$\frac{2}{5} + \frac{2}{4} =$$

D.

$$\frac{1}{5} + \frac{1}{2} =$$

D.

$$\frac{2}{12} + \frac{2}{4} =$$

D.

$$\frac{5}{9} + \frac{1}{3} =$$

D.

$$\frac{1}{4} + \frac{5}{8} =$$

D.

$$\frac{2}{12} + \frac{2}{3} =$$

E.

$$\frac{4}{2} + \frac{5}{3} =$$

E.

$$\frac{4}{5} + \frac{5}{4} =$$

E.

$$\frac{5}{2} + \frac{7}{4} =$$

E.

$$\frac{6}{5} + \frac{5}{3} =$$

E.

$$\frac{11}{4} + \frac{10}{8} =$$

E.

$$\frac{8}{7} + \frac{9}{5} =$$

E.

$$\frac{7}{4} + \frac{5}{8} =$$

E.

$$\frac{11}{8} + \frac{3}{2} =$$

E.

$$\frac{13}{5} + \frac{2}{4} =$$

E. $\frac{12}{10} + \frac{10}{4} =$

F.

$$1\frac{1}{2} + 1\frac{7}{8} =$$

F.

$$\frac{2}{5} + 4\frac{1}{4} =$$

F.

$$7\frac{1}{2} + 3\frac{1}{5} =$$

F.

$$1\frac{5}{6} + 1\frac{2}{4} =$$

F.

$$\frac{7}{8} + 2\frac{1}{2} =$$

F.

$$1\frac{4}{10} + 1\frac{2}{5} =$$

F.

$$7\frac{3}{8} + 2\frac{5}{12} =$$

F.

$$\frac{2}{3} + 3\frac{1}{9} =$$

F.

$$\frac{1}{2} + 2\frac{5}{6} =$$

F.

$$1\frac{2}{6} + 4\frac{5}{12} =$$

G.

$$\begin{array}{r} 0.3 \\ + 0.1 \\ \hline \end{array}$$

G.

$$\begin{array}{r} 1.5 \\ + 2.2 \\ \hline \end{array}$$

G.

$$\begin{array}{r} 3.2 \\ + 0.3 \\ \hline \end{array}$$

G.

$$\begin{array}{r} 2.5 \\ + 4.2 \\ \hline \end{array}$$

G.

$$\begin{array}{r} 0.1 \\ + 4.1 \\ \hline \end{array}$$

G.

3.3

+ 4.6

G.

$$\begin{array}{r} 0.8 \\ + 2.1 \\ \hline \end{array}$$

G.

$$\begin{array}{r} 1.6 \\ + 4.1 \\ \hline \end{array}$$

G.

$$\begin{array}{r} 6.3 \\ + 2.1 \\ \hline \end{array}$$

G.

$$\begin{array}{r} 3.1 \\ + 1.8 \\ \hline \end{array}$$

G.

$$\begin{array}{r} 5.8 \\ + 4.1 \\ \hline \end{array}$$

G.

$$\begin{array}{r} 9.2 \\ + 0.4 \\ \hline \end{array}$$

G.

$$\begin{array}{r} 1.7 \\ + 6.2 \\ \hline \end{array}$$

G.

$$\begin{array}{r} 0.7 \\ + 0.2 \\ \hline \end{array}$$

G.

5.3

+ 4.4



G.

$$\begin{array}{r} 6.1 \\ + 3.2 \\ \hline \end{array}$$

G.

$$\begin{array}{r} 5.4 \\ + 0.4 \\ \hline \end{array}$$

G.

$$\begin{array}{r} 0.3 \\ + 0.6 \\ \hline \end{array}$$

G.

$$\begin{array}{r} 2.2 \\ + 7.0 \\ \hline \end{array}$$

G.

$$\begin{array}{r} 6.4 \\ + 3.3 \\ \hline \end{array}$$

H.

$$\begin{array}{r} 4.2 \\ + 2.8 \\ \hline \end{array}$$

H.

2.3

+ 6.7

H.

$$\begin{array}{r} 1.5 \\ + 5.6 \\ \hline \end{array}$$

H.

$$\begin{array}{r} 2.1 \\ + 3.9 \\ \hline \end{array}$$

H.

$$\begin{array}{r} 4.8 \\ + 3.6 \\ \hline \end{array}$$

H.

$$\begin{array}{r} 4.5 \\ + 3.9 \\ \hline \end{array}$$

H.

$$\begin{array}{r} 2.9 \\ + 5.4 \\ \hline \end{array}$$

H.

$$\begin{array}{r} 6.2 \\ + 2.9 \\ \hline \end{array}$$

H.

$$\begin{array}{r} 1.1 \\ + 6.9 \\ \hline \end{array}$$

H.

$$\begin{array}{r} 4.2 \\ + 3.9 \\ \hline \end{array}$$

H.

$$\begin{array}{r} 1.8 \\ + 7.4 \\ \hline \end{array}$$

H.

$$\begin{array}{r} 5.1 \\ + 2.9 \\ \hline \end{array}$$

H.

$$\begin{array}{r} 3.5 \\ + 5.7 \\ \hline \end{array}$$

H.

4.1

+ 4.9

H.

$$\begin{array}{r} 5.6 \\ + 4.7 \\ \hline \end{array}$$

H.

$$\begin{array}{r} 6.9 \\ + 3.2 \\ \hline \end{array}$$

H.

$$\begin{array}{r} 8.8 \\ + 1.6 \\ \hline \end{array}$$

H.

$$\begin{array}{r} 2.2 \\ + 7.8 \\ \hline \end{array}$$

H.

$$\begin{array}{r} 3.5 \\ + 6.6 \\ \hline \end{array}$$

H.

$$\begin{array}{r} 1.2 \\ + 6.8 \\ \hline \end{array}$$

1.

$$\begin{array}{r} 0.73 \\ + 0.21 \\ \hline \end{array}$$

1.

$$\begin{array}{r} 1.46 \\ + 3.32 \\ \hline \end{array}$$

1.

2.58

+ 6.11



l.

$$\begin{array}{r} 9.82 \\ + 0.01 \\ \hline \end{array}$$

1.

$$\begin{array}{r} 0.31 \\ + 8.22 \\ \hline \end{array}$$

l.

$$\begin{array}{r} 1.50 \\ + 2.46 \\ \hline \end{array}$$

1.

2.31

+ 1.60



1.

$$\begin{array}{r} 7.31 \\ + 2.47 \\ \hline \end{array}$$

1.

$$\begin{array}{r} 9.13 \\ + 0.60 \\ \hline \end{array}$$

l.

$$\begin{array}{r} 12.46 \\ + 1.10 \\ \hline \end{array}$$

I.

23.20

+ 6.04



l.

$$\begin{array}{r} 1.71 \\ + 4.10 \\ \hline \end{array}$$

1.

2.35

+ 4.22



l.

$$\begin{array}{r} 0.88 \\ + 1.01 \\ \hline \end{array}$$

l.

3.63

+ 1.21



I.

$$\begin{array}{r} 10.13 \\ + 10.26 \\ \hline \end{array}$$

1.

$$\begin{array}{r} 9.34 \\ + 2.44 \\ \hline \end{array}$$

1.

$$\begin{array}{r} 5.60 \\ + 1.22 \\ \hline \end{array}$$

1.

$$\begin{array}{r} 6.31 \\ + 3.08 \\ \hline \end{array}$$

l.

$$\begin{array}{r} 10.33 \\ + 0.55 \\ \hline \end{array}$$

J.

$$\begin{array}{r} 2.56 \\ + 3.45 \\ \hline \end{array}$$

J.

$$\begin{array}{r} 5.24 \\ + 1.37 \\ \hline \end{array}$$

J.

$$\begin{array}{r} 5.45 \\ + 3.78 \\ \hline \end{array}$$

J.

$$\begin{array}{r} 3.67 \\ + 5.25 \\ \hline \end{array}$$

J.

$$\begin{array}{r} 6.14 \\ + 1.47 \\ \hline \end{array}$$

J.

$$\begin{array}{r} 4.25 \\ + 2.25 \\ \hline \end{array}$$

J.

$$\begin{array}{r} 4.71 \\ + 3.89 \\ \hline \end{array}$$

J.

$$\begin{array}{r} 1.52 \\ + 3.77 \\ \hline \end{array}$$

J.

2.84

+ 6.16



J.

$$\begin{array}{r} 14.80 \\ + 6.96 \\ \hline \end{array}$$

J.

$$\begin{array}{r} 7.83 \\ + 6.99 \\ \hline \end{array}$$

J.

$$\begin{array}{r} 8.95 \\ + 9.80 \\ \hline \end{array}$$

J.

$$\begin{array}{r} 12.80 \\ + 46.93 \\ \hline \end{array}$$

J.

$$\begin{array}{r} 3.14 \\ + 1.99 \\ \hline \end{array}$$

J.

$$\begin{array}{r} 7.21 \\ + 4.66 \\ \hline \end{array}$$

J.

$$\begin{array}{r} 5.44 \\ + 2.08 \\ \hline \end{array}$$

J.

$$\begin{array}{r} 9.66 \\ + 1.67 \\ \hline \end{array}$$

J.

8.33

+ 1.92

J.

$$\begin{array}{r} 42.12 \\ + 10.09 \\ \hline \end{array}$$

J.

$$\begin{array}{r} 6.87 \\ + 2.33 \\ \hline \end{array}$$

K.

$$\begin{array}{r} 30.15 \\ + 2.6 \\ \hline \end{array}$$

K.

$$\begin{array}{r} 1.5 \\ + 2.49 \\ \hline \end{array}$$

K.

$$\begin{array}{r} 14.58 \\ + 1.4 \\ \hline \end{array}$$

K.

$$\begin{array}{r} 10.2 \\ + 5.73 \\ \hline \end{array}$$

K.

$$\begin{array}{r} 5.4 \\ + .54 \\ \hline \end{array}$$

K.

8.3

+

.91



K.

4.6

+

.64



K.

$$\begin{array}{r} 9.38 \\ + .19 \\ \hline \end{array}$$

K.

$$\begin{array}{r} 10.21 \\ + 5.6 \\ \hline \end{array}$$

K.

$$\begin{array}{r} 1.9 \\ + 2.01 \\ \hline \end{array}$$

K.

$$\begin{array}{r} 17.72 \\ + 12.58 \\ \hline \end{array}$$

K.

$$\begin{array}{r} 42.1 \\ + 17.96 \\ \hline \end{array}$$

K.

$$\begin{array}{r} 8.3 \\ + 9.31 \\ \hline \end{array}$$

K.

$$\begin{array}{r} 9.0 \\ + 8.12 \\ \hline \end{array}$$

K.

$$\begin{array}{r} 9.17 \\ + 2.7 \\ \hline \end{array}$$

K.

3.46

+

1.6



K.

$$\begin{array}{r} 4.9 \\ + 9.23 \\ \hline \end{array}$$

K.

$$\begin{array}{r} 15.5 \\ + 12.22 \\ \hline \end{array}$$

K.

$$\begin{array}{r} 17.5 \\ + 8.83 \\ \hline \end{array}$$

K.

$$\begin{array}{r} 9.2 \\ + 6.75 \\ \hline \end{array}$$

Module 6:

Addition of Rational Numbers

Vocabulary Cards

add/addition

addend

algorithm

computation

decimal

denominator

equal sign

equivalent

fraction

hundredths

improper fraction

join

least common multiple

mixed number

multiple

numerator

ones

plus sign

regroup/trade/exchange

sum

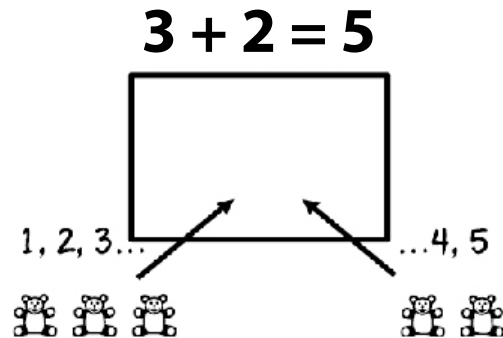
tenths

together

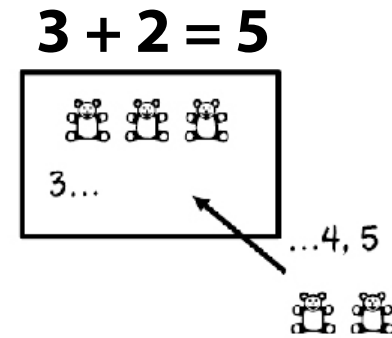
add/addition

To put amounts together to find the sum or to increase a set.

To put amounts together



To increase a set



addend

Any numbers that are added together.

$$6 + 2 = 8$$

6 and **2** are addends

algorithm

A procedure or description of steps that can be used to solve a problem.

computation

The action used to solve a problem.

decimal

A number based on powers of ten.

34.107
tens ones tenths thousandths

denominator

The term in a fraction that tells the number of equal parts in a whole.

$2 / 3$ $\frac{2}{3}$ In these fractions, **3** is the denominator.

equal sign

The symbol that tells you that two sides of an equation are the same, balanced, or equal.

$$12 + 8 = 20$$

= is the **equal sign**

equivalent

Two numbers that have the same value.

$$\frac{1}{4} = \frac{2}{8} \qquad \frac{2}{3} = \frac{8}{12}$$

fraction

A number representing part of a whole or set.

$$\frac{3}{6} \quad \frac{10}{12} \quad \frac{8}{3}$$

hundredths

The digit in representing $\frac{1}{100}$.

In the number 4.23, 3 is in the hundredths place.

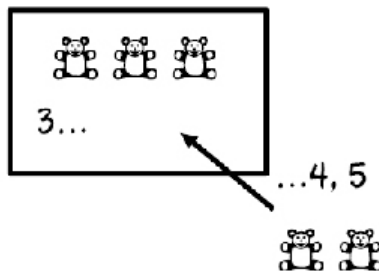
improper fraction

Any fraction in which the numerator is greater than or equal to the denominator.

$$\frac{9}{4} \quad \frac{17}{12} \quad \frac{10}{3}$$

join

To add to an existing set.



least common multiple

The common multiple with the least value.

$$6: 6, 12, 18, 24, 30$$

$$8: 8, 16, 24, 32, 40$$

With multiples of 6 and 8, the **least common multiple** is 24.

mixed number

A whole number and a fraction combined.

$$1\frac{1}{6}$$

$$4\frac{5}{12}$$

$$12\frac{4}{3}$$

multiple

The product of a number and any integer.

4: 4, 8, 12, 16, 20

numerator

The term in a fraction that tells how many parts in a fraction.

2 / **3**

2
—
3

In these fractions, **2** is the numerator.

ones

The digit representing 1.

In the number 4.23, 4 is in the ones place.

plus sign

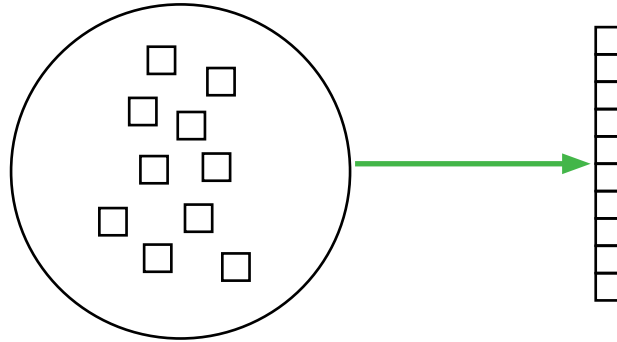
The symbol that tells you to add.

$$5 + 4 = 9$$

+ is the plus sign

regroup/trade/exchange

The process of exchanging 10 ones for 1 ten, 10 tens for 1 hundred, 10 hundreds for 1 thousand, etc.



sum

The result of adding two or more numbers.

$$7 + 2 + 1 = 10$$

10 is the **sum**

tenths

The digit in representing $\frac{1}{10}$.

In the number 4.23, 2 is in the tenths place.

together

To combine sets or numbers.

