



Instructional Routines for Mathematics Intervention

The purpose of these mathematics instructional routines is to provide educators with materials to use when providing intervention to students who experience difficulty with mathematics. The routines address content included in the grades 2-8 Texas Essential Knowledge and Skills (TEKS). There are 23 modules that include routines and examples – each focused on different mathematical content. Each of the 23 modules include vocabulary cards and problem sets to use during instruction. These materials are intended to be implemented explicitly with the aim of improving mathematics outcomes for students.

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Instructional Routines for Mathematics Intervention

MODULE 9

Subtraction of Rational Numbers



Module 9: Subtraction of Rational Numbers

Mathematics Routines

A. Important Vocabulary with Definitions

Term	Definition
algorithm	A procedure or description of steps that can be used to solve a problem.
compare	To find the difference between two sets.
computation	The action used to solve a problem.
decimal	A number based on powers of ten.
denominator	The term in a fraction that tells the number of equal parts in a whole.
difference	The result of subtracting one number from another number.
equal sign	The symbol that tells you that two sides of an equation are the same, balanced, or equal.
equivalent	Two numbers that have the same value.
fraction	A number representing part of a whole or set.
hundredths	The digit in representing $\frac{1}{100}$.
improper fraction	Any fraction in which the numerator is greater than the denominator.
least common multiple	The common multiple with the least value.
minuend	The number from which another number is subtracted.
minus sign	The symbol that tells you to subtract.
mixed number	A whole number and a fraction combined.
multiple	The product of a number and any integer.
numerator	The term in a fraction that tells how many parts of a fraction.
ones	The digit representing 1.
regroup/trade/exchange	The process of exchanging 10 ones for 1 ten, 10 tens for 1 hundred, 10 hundreds for 1 thousand, etc.
separate	To start with a set and take away from that set.
subtract/subtraction	To compare two sets or to separate from a set.
subtrahend	The number to be subtracted.
tenths	The digit in representing $\frac{1}{10}$.

B. Background Information

Background Information:

In this module, we focus on subtraction with fractions and decimals. As you focus on computation of rational numbers, continue to emphasize subtraction as separating and subtraction as comparing because students will see these concepts within word problems.

For subtraction of fractions, we recommend using several models of fractions to help students understand concepts related to subtraction of fractions. We also recommend demonstrating several algorithms for subtraction of decimals. Every student should develop efficiency with strategies for subtraction of fractions and decimals. In the following sections, we provide examples of (1) subtraction of fractions – like denominators, (2) subtraction of fractions – unlike denominators, (3) subtraction of decimals with the traditional algorithm, and (4) subtraction of decimals with the adding up algorithm.

C. Routines and Examples

(1) Subtraction of Fractions – Like Denominators

Routine

Materials:

- [Module 9 Problem Sets](#)
- [Module 9 Vocabulary Cards](#)
 - If necessary, review Vocabulary Cards before teaching
- A hands-on tool or manipulative like fraction tiles or two-color counters
 - Note that drawings can be used alongside or instead of manipulatives

ROUTINE WITH MANIPULATIVES

- Teacher** Let's work on subtraction. What does it mean to subtract?
- Students** To separate from a set or to compare.
- Teacher** Subtraction means to separate from a set or to compare two numbers. Look at this problem.
(Show problem.)
- Teacher** First, I see a minus sign (point). The minus sign tells us to subtract. What does the minus sign mean?
- Students** To subtract.
- Teacher** Let's do this problem with fraction tiles.
(Move fraction tiles to workspace.)
- Teacher** First, our minuend is __. What's the minuend?
- Students** __.
- Teacher** Let's show this minuend using the fraction tiles.
(Show fraction part compared to whole.)

Teacher What fraction?
 Students ___.
Teacher Now, our subtrahend is ___. What's our subtrahend?
 Students ___.
Teacher We'll subtract the subtrahend. When working with fractions, I like to show the subtrahend to know the quantity we will separate from the minuend. Let's show the subtrahend over here.
 (Show fraction part compared to whole.)
Teacher What fraction?
 Students ___.
Teacher Let's subtract. When subtracting fractions, first we want to determine whether the denominators are like or unlike. Are the denominators like or the same?
 Students Yes.
Teacher The denominators are the same. When the denominators are the same, we can go ahead and subtract. So, let's look at our subtrahend. We want to subtract the subtrahend from the minuend. How many ___ one-___ parts do we subtract?
 Students ___ one-___ parts.
Teacher Let's subtract ___ one-___ parts.
 Students (Subtract from the minuend set.)
Teacher So, we now have __, __, __, ... one-___ parts remaining. How many parts?
 Students ___.
Teacher When you have ___ minus __, the difference is __. What's the difference?
 Students ___.
Teacher ___ minus ___ equals __. Let's say that together.
 Students ___ minus ___ equals __.
Teacher So, if you have a set of ___ and a set of __, when you subtract (or separate) the sets, the difference is __. ___ minus ___ equals __. Let's review. What's a minuend?
 Students The number from which another is subtracted.
Teacher What's a subtrahend?
 Students The number to be subtracted.
Teacher What's a difference?
 Students The result of subtracting a subtrahend from a minuend.
Teacher What does it mean to separate?
 Students To take away.
Teacher How could you explain separating to a friend?
 Students We started with a fraction showing fraction tiles. We checked whether the denominators were the same. Then, we separated ___ one-___ parts from the minuend to learn the difference.
Teacher What's another way we could have solved this problem?
 Students We could have compared two sets.

ROUTINE WITHOUT MANIPULATIVES

- Teacher** Let's work on subtraction. What does it mean to subtract?
- Students** To separate or compare.
- Teacher** Subtraction means to separate from a set or to compare two sets. Look at this problem.
(Show problem.)
- Teacher** First, I see a minus sign (point). The minus sign tells us to subtract. What does the minus sign mean?
- Students** To subtract.
- Teacher** Let's do this problem with our pencil. Our minuend is __. What's our minuend?
- Students** __.
- Teacher** Our subtrahend is __. What's our subtrahend?
- Students** __.
- Teacher** So, we have __ minus __. Let's subtract by separating. What does separating mean?
- Students** To remove some from a set.
- Teacher** Yes. Let's subtract, or separate, the subtrahend from the minuend. What do we subtract?
- Students** The subtrahend from the minuend.
- Teacher** Now, the parts of the fractions are the numerators. When we subtract fractions, first we want to determine whether the denominators are like or unlike. Are the denominators like or the same?
- Students** Yes.
- Teacher** The denominators are the same. When the denominators are the same, we can go ahead and subtract. The denominator, __, will not change when we subtract the fractions. Let's go ahead and write the denominator for our difference.
(Write denominator.)
- Teacher** Now, we want to subtract the parts or numerator of the subtrahend from the minuend. That means we have to subtract __ one-__ parts from __ one-__ parts. What do we subtract?
- Students** We subtract the parts or numerators of the fractions.
- Teacher** Let's subtract the parts. What's __ minus __?
- Students** __.
- Teacher** Let's write the parts we subtracted.
(Write parts.)
- Teacher** When you have __ minus __, the difference is __. What's the difference?
- Students** __.
- Teacher** __ minus __ equals __. Let's say that together.
- Students** __ minus __ equals __.
- Teacher** So, if you have a set of __ and a set of __, when you subtract (or separate) the subtrahend from the minuend, the difference is __. __ minus __ equals __.
Let's review. What's a minuend?

Students The number from which another is subtracted.

Teacher **What's a subtrahend?**

Students The number to be subtracted.

Teacher **What's a difference?**

Students The result of subtracting a subtrahend from a minuend.

Teacher **What does it mean to separate?**

Students To take away.

Teacher **How could you explain separating to a friend?**

Students We checked whether the denominators were the same. Then, we subtracted the parts of the subtrahend from the parts of the minuend to learn the difference between two numbers.

Teacher **What's another way we could have solved this problem?**

Students We could have compared two sets.

Example

$$\frac{5}{6} - \frac{3}{6} = \frac{2}{6}$$

EXAMPLE WITH MANIPULATIVES

Teacher **Let's work on subtraction. What does it mean to subtract?**

Students To separate or compare.

Teacher **Subtraction means to separate from a set or compare two sets. Look at this problem.**
(Show problem.)

Teacher **First, I see a minus sign (point). The minus sign tells us to subtract. What does the minus sign mean?**

Students To subtract.

Teacher **Let's do this problem with fraction tiles.**
(Move fraction tiles to workspace.)

Teacher **Our minuend is $\frac{5}{6}$. What's our minuend?**

Students $\frac{5}{6}$.

Teacher **Let's show this minuend by showing the fraction.**
(Show 5 one-sixth parts compared to a whole.)

Teacher **What fraction?**

Students $\frac{5}{6}$.

Teacher **Our subtrahend is $\frac{3}{6}$. What's our subtrahend?**

Students $\frac{3}{6}$.

Teacher **Let's show the subtrahend by showing the fraction.**
(Show 3 one-sixth parts compared to a whole.)

Teacher What fraction?

Students $\frac{3}{6}$.

Teacher So, we have $\frac{5}{6}$ minus $\frac{3}{6}$. Let's subtract the subtrahend from the minuend. What does subtracting mean?

Students To separate or compare.

Teacher Let's subtract, or separate, the parts of the fractions. The parts of the fractions represent the numerators. When subtracting fractions, first we want to determine whether the denominators are like or unlike. Are the denominators like or the same?

Students Yes.

Teacher Both denominators are 6. The denominators are the same or like denominators. When the denominators are the same, we can go ahead and subtract. Second, we want to subtract the numerators, or parts, or the subtrahend from the minuend. That means we have to subtract 3 one-sixth parts from 5 one-sixth parts. What do we subtract?

Students We subtract the parts or numerators of the fraction.

Teacher Let's subtract the 3 one-sixth parts from the 5 one-sixth parts. I'm not going to touch the subtrahend. Instead, I separate, or take away, 3 one-sixth parts from the minuend.
(Subtract parts, compare to whole.)

Teacher So, we now have 1, 2 one-sixth parts. How many parts?

Students 2 one-sixth parts.

Teacher When you have $\frac{5}{6}$ minus $\frac{3}{6}$, the difference is $\frac{2}{6}$. What's the difference?

Students $\frac{2}{6}$.

Teacher $\frac{5}{6}$ minus $\frac{3}{6}$ equals $\frac{2}{6}$. Let's say that together.

Students $\frac{5}{6}$ minus $\frac{3}{6}$ equals $\frac{2}{6}$.

Teacher So, if you have a set of $\frac{5}{6}$ and you separate $\frac{3}{6}$, when you subtract the parts or numerators of the subtrahend from the minuend, the difference is $\frac{2}{6}$. $\frac{5}{6}$ minus $\frac{3}{6}$ equals $\frac{2}{6}$. Let's review. What's a minuend?

Students The number from which another is subtracted.

Teacher What's a subtrahend?

Students The number to be subtracted.

Teacher What's a difference?

Students The result of subtracting a subtrahend from a minuend.

Teacher What does it mean to separate?

Students To take away.

Teacher How could you explain separating to a friend?

Students We showed the minuend with fraction tiles and showed the subtrahend with fraction tiles. Then, we subtracted 3 one-sixth parts from 5 one-sixth parts. The difference was two-sixths.

Teacher **What's another way we could have solved this problem?**

Students We could have compared two sets.

(2) Subtraction of Fractions – Unlike Denominators

Routine

Materials:

- [Module 9 Problem Sets](#)
- [Module 9 Vocabulary Cards](#)
 - If necessary, review Vocabulary Cards before teaching
- A hands-on tool or manipulative like fraction tiles or two-color counters
 - Note that drawings can be used alongside or instead of manipulatives

ROUTINE WITH MANIPULATIVES

Teacher **Let's work on subtraction. What does it mean to subtract?**

Students To separate or compare.

Teacher **Subtraction means to separate from a set or compare two sets. Look at this problem.**
(Show problem.)

Teacher **First, I see a minus sign (point). The minus sign tells us to subtract. What does the minus sign mean?**

Students To subtract.

Teacher **Let's do this problem with two-color counters.**
(Move two-color counters to workspace.)

Teacher **Our minuend is __. What's our minuend?**

Students __.

Teacher **Let's show this minuend by showing the fraction.**
(Show set compared to whole with white/yellow counters representing numerator and red counters representing denominator.)

Teacher **What fraction?**

Students __.

Teacher **Our subtrahend is __. What's our subtrahend?**

Students __.

Teacher **Let's show the subtrahend by showing the fraction.**
(Show set compared to whole with white/yellow counters representing numerator and red counters representing denominator.)

Teacher **What fraction?**

Students __.

Teacher So, we have __ minus __. Let's subtract by separating. What does separating mean?

Students To take away from a set.

Teacher Yes. Let's separate, or take away, the subtrahend from the minuend. Remember, the parts of the fractions represent the numerators. When subtracting fractions, first we want to determine whether the denominators are like or unlike. You might also say common or uncommon denominators. Are the denominators the same or alike?

Students No.

Teacher The denominators are not the same. To subtract, we need to subtract parts or numerators with the same denominator. When the denominators are unlike, the parts or numerators do not have the same value. So, we will work to make the fractions have like denominators. Why do we want to subtract fractions with like denominators?

Students So, we can subtract the parts or numerators of the fractions.

Teacher To do this, let's write the first five multiples of each denominator. The minuend has a denominator of __, so let's write the first five multiples of __. (Write multiples as __, __, __, __, __.)

Teacher What are the multiples of __? Say them with me.

Students __, __, __, __, __.

Teacher The subtrahend has a denominator of __, so let's write the first five multiples of __. (Write multiples as __, __, __, __, __.)

Teacher What are the multiples of __? Say them with me.

Students __, __, __, __, __.

Teacher Great. Let's determine the least common multiple of the two fractions. What is the multiple with the least value that you see on both lists of multiples?

Students __.

Teacher So, __ is the least common multiple. Say that with me.

Students Least common multiple.

Teacher Sometimes we call the least common multiple the LCM. What do we call the least common multiple?

Students LCM.

Teacher The least common multiple, or LCM, helps us to determine the common denominator for the two fractions. What does the LCM help with?

Students Finding a common denominator for the two fractions.

Teacher The minuend has a denominator of __.

OPTION 1: This is the original denominator. We don't have to do anything to this fraction.

OPTION 2: This is not the original denominator. We need to convert the fraction from a denominator of __ to a denominator of __.

What do we need to do?

Students **OPTION 1:** We don't have to change the denominator.
OPTION 2: We need to convert the fraction to a denominator of __.

Teacher **OPTION 2:** To convert the fraction to a denominator of __, I determine how many groups of __ (original denominator) I need to make __ (common denominator). I see I need to make __, __, __ groups of __ (original denominator). How many groups?

Students __.

Teacher So, I make __ groups of __ with the two-color counters. That means I iterate or copy the original fraction __ times. What does it mean to iterate?

Students To copy.

Teacher Our new fraction is __. Is __ (original fraction) equivalent to __ (fraction with common denominator)?

Students Yes.

Teacher How do you know the fractions are equivalent?

Students The fractions have the same value. They are equivalent.

Teacher So, we converted the minuend to a common denominator. Let's do the same with the subtrahend. What's the subtrahend?

__.

Teacher The subtrahend has a denominator of __.

OPTION 1: This is the original denominator. We don't have to do anything to this fraction.

OPTION 2: This is not the original denominator. We need to convert the fraction from a denominator of __ to a denominator of __.

What do we need to do?

Students **OPTION 1:** We don't have to change the denominator.

OPTION 2: We need to convert the fraction to a denominator of __.

Teacher **OPTION 2:** To convert the fraction to a denominator of __, I determine how many groups of __ (original denominator) I need to make __ (common denominator). I see I need to make __, __, __ groups of __ (original denominator). How many groups?

Students __.

Teacher We make __ groups of __ with the two-color counters. That means I iterate or copy the original fraction __ times. How many times?

Students __.

Teacher Let's check our work. Is __ (original fraction) equivalent to __ (fraction with common denominator)?

Students Yes.

Teacher How do you know the fractions are equivalent?

Students The fractions have the same value. They are equivalent.

Teacher Now that we have common denominators, we want to subtract the parts or numerators of subtrahend from the minuend. That means we have to subtract __ one-__ parts from __ one-__ parts. What do we subtract?

Students We subtract the parts or numerators of the fractions.

Teacher Let's subtract the numerators. I like to keep my subtrahend set where it is and only subtract from the minuend set. We need to subtract the red one-__ parts. How many parts do we have to subtract?

Students __.

Teacher We subtract __ one-__ parts. I subtract by turning over (to yellow) __ one-__ parts. How many parts?

Students __.

Teacher When you have __ minus __, the difference is __. What's the difference?

Students __.

Teacher __ minus __ equals __. Let's say that together.

Students __ minus __ equals __.

Teacher So, if you have a set of __ and subtract a set of __, the difference is __. __ minus __ equals __. Let's review. What's a minuend?

Students The number from which another is subtracted.

Teacher What's a subtrahend?

Students The number to be subtracted.

Teacher What's a difference?

Students The result of subtracting a subtrahend from a minuend.

Teacher What does it mean to separate?

Students To take away.

Teacher How could you explain separating to a friend?

Students We showed the minuend and the subtrahend. Then, we determined the common denominator using the LCM. After converting the fractions to common denominators, we subtracted the subtrahend parts from the minuend parts to learn of the difference.

Teacher What's another way we could have solved this problem?

Students We could have compared two sets.

ROUTINE WITHOUT MANIPULATIVES

Teacher Let's work on subtraction. What does it mean to subtract?

Students To separate or compare.

Teacher Subtraction means to separate from a set or compare two sets. Look at this problem.

(Show problem.)

Teacher First, I see a minus sign (point). The minus sign tells us to subtract. What does the minus sign mean?

Students To subtract.

Teacher Our minuend is __. What's our minuend?

Students __.

Teacher Our subtrahend is __. What's our subtrahend?

Students __.

Teacher So, we have __ minus __. Let's subtract by separating. What does separating mean?

Students To take away.

Teacher Yes. Let's separate, or take away, the subtrahend from the minuend. Remember, the parts of fractions represent the numerator. What do you subtract?

Students The parts or numerators of the fractions.

Teacher When subtracting fractions, first we want to determine whether the denominators are like or unlike. You might also say common or uncommon denominators. Are the denominators the same or alike?

Students No.

Teacher The denominators are not the same. To subtract, we need to subtract the parts or numerators with the same value. When the denominators are unlike, the parts or numerators do not represent the same value. So, we will work to make the fractions have like denominators. Why do we want to subtract fractions with like denominators?

Students So we can subtract the parts or numerators of the fractions.

Teacher To do this, let's write the first five multiples of each denominator. The minuend has a denominator of __, so let's write the first five multiples of __. (Write multiples as __, __, __, __, __.)

Teacher What are the multiples of __? Say them with me.

Students __, __, __, __, __.

Teacher The subtrahend has a denominator of __, so let's write the first five multiples of __. (Write multiples as __, __, __, __, __.)

Teacher What are the multiples of __? Say them with me.

Students __, __, __, __, __.

Teacher Great. Let's determine the least common multiple of the two fractions. What is the multiple with the least value that you see on both lists of multiples?

Students __.

Teacher So, __ is the least common multiple. Say that with me.

Students Least common multiple.

Teacher Sometimes we call the least common multiple the LCM. What do we call the least common multiple?

Students LCM.

Teacher The least common multiple, or LCM, helps us determine the common denominator for the two fractions. What does the LCM help with?

Students Finding a common denominator for the two fractions.

Teacher The minuend has a denominator of __.

OPTION 1: This is the original denominator. We don't have to do anything to this fraction.

OPTION 2: This is not the original denominator. We need to convert the fraction from a denominator of __ to a denominator of __.

What do we need to do?

Students *OPTION 1:* We don't have to change the denominator.
OPTION 2: We need to convert the fraction to a denominator of ____.

Teacher ***OPTION 2:*** To convert the fraction to a denominator of __, I determine how many groups of __ (original denominator) I need to make __ (common denominator). I see I need to make __, __, __ groups of __ (original denominator). How many groups?
 ____.

Students ____.
 Teacher So, I multiply the denominator times __ and the numerator times __. Let's multiply the denominator first. __ (original denominator) times __ is what?
 ____.

Students ____.
 Teacher That's right. __ times __ equals __. Our new denominator is __. What's our new denominator?
 ____.

Students ____.
 Teacher Now, let's multiply the numerator times __. __ (original numerator) times __ is what?
 ____.

Students ____.
 Teacher Yes. __ times __ equals __. Our new numerator is __. What's the new numerator?
 ____.

Students ____.
 Teacher Let's check our work. Is __ (original fraction) equivalent to __ (fraction with common denominator)? How do you know the fractions are equivalent?
 The fractions have the same value. They are equivalent.

Students ____.
 Teacher So, we converted the minuend to a common denominator. Let's do the same with the subtrahend. What's the subtrahend?
 ____.

Teacher The subtrahend has a denominator of ____.
OPTION 1: This is the original denominator. We don't have to do anything to this fraction.
OPTION 2: This is not the original denominator. We need to convert the fraction from a denominator of __ to a denominator of __.
 What do we need to do?

Students *OPTION 1:* We don't have to change the denominator.
OPTION 2: We need to convert the fraction to a denominator of ____.

Teacher ***OPTION 2:*** To convert the fraction to a denominator of __, I determine how many groups of __ (original denominator) I need to make __ (common denominator). I see I need to make __, __, __ groups of __ (original denominator). How many groups?
 ____.

Students ____.
 Teacher So, I multiply the denominator times __ and the numerator times __. Let's multiply the denominator first. __ (original denominator) times __ is what?
 ____.

Students ____.

Teacher **That's right. $\frac{_}{_}$ times $\frac{_}{_}$ equals $\frac{_}{_}$. Our new denominator is $\frac{_}{_}$. What's our new denominator?**

Students $\frac{_}{_}$.

Teacher **Now, let's multiply the numerator times $\frac{_}{_}$. $\frac{_}{_}$ (original numerator) times $\frac{_}{_}$ is what?**

Students $\frac{_}{_}$.

Teacher **Yes. $\frac{_}{_}$ times $\frac{_}{_}$ equals $\frac{_}{_}$. Our new numerator is $\frac{_}{_}$. What's the new numerator?**

Students $\frac{_}{_}$.

Teacher **Let's check our work. Is $\frac{_}{_}$ (original fraction) equivalent to $\frac{_}{_}$ (fraction with common denominator)? How do you know the fractions are equivalent?**

Students Yes.

Teacher **How do you know the fractions are equivalent?**

Students The fractions have the same value. They are equivalent.

Teacher **Now that we have common denominators, we want to subtract the parts or numerator of the subtrahend from the minuend. That means we have to subtract $\frac{_}{_}$ one- $\frac{_}{_}$ parts from $\frac{_}{_}$ one- $\frac{_}{_}$ parts. What do we subtract?**

Students We subtract the parts of the fractions.

Teacher **Let's subtract the parts or numerators.**
(Subtract parts, compare to whole.)

Teacher **So, we now have $\frac{_}{_}$, $\frac{_}{_}$, $\frac{_}{_}$, ... one- $\frac{_}{_}$ parts. How many parts?**

Students $\frac{_}{_}$.

Teacher **When you have $\frac{_}{_}$ minus $\frac{_}{_}$, the difference is $\frac{_}{_}$. What's the difference?**

Students $\frac{_}{_}$.

Teacher **$\frac{_}{_}$ minus $\frac{_}{_}$ equals $\frac{_}{_}$. Let's say that together.**

Students $\frac{_}{_}$ minus $\frac{_}{_}$ equals $\frac{_}{_}$.

Teacher **So, if you have a set of $\frac{_}{_}$ and subtract a set of $\frac{_}{_}$, the difference is $\frac{_}{_}$. $\frac{_}{_}$ minus $\frac{_}{_}$ equals $\frac{_}{_}$. Let's review. What's a minuend?**

Students The number from which another is subtracted.

Teacher **What's a subtrahend?**

Students The number to be subtracted.

Teacher **What's a difference?**

Students The result of subtracting a subtrahend from a minuend.

Teacher **What does it mean to separate?**

Students To take away.

Teacher **How could you explain separating to a friend?**

Students After determining a common denominator, we subtracted the subtrahend from the minuend to learn the difference.

Teacher **What's another way we could have solved this problem?**

Students We could have compared two sets.

Example

$$\frac{7}{8} - \frac{1}{4} = \frac{5}{8}$$

EXAMPLE WITH MANIPULATIVES

- Teacher** Let's work on subtraction. What does it mean to subtract?
- Students** To separate or compare.
- Teacher** Subtraction means to separate from a set or compare two sets. Look at this problem.
(Show problem.)
- Teacher** First, I see a minus sign (point). The minus sign tells us to subtract. What does the minus sign mean?
- Students** To subtract.
- Teacher** Let's do this problem with two-color counters.
(Move two-color counters to workspace.)
- Teacher** Our minuend is $\frac{7}{8}$. What's our minuend?
- Students** $\frac{7}{8}$.
- Teacher** Let's show this minuend by showing the fraction. First, we have a denominator of 8, so let's show 8 yellow counters. How many?
- Students** 8.
- Teacher** Then, we need to show 7 of the 8 parts as red to show $\frac{7}{8}$. How many should we make red?
- Students** 7.
- Teacher** What fraction?
- Students** $\frac{7}{8}$.
- Teacher** Our subtrahend is $\frac{1}{4}$. What's our subtrahend?
- Students** $\frac{1}{4}$.
- Teacher** Let's show the subtrahend by showing the fraction. First, we have a denominator of 4, so let's show 4 yellow counters. How many?
- Students** 4.
- Teacher** Then, we need to show 1 of the 4 parts as red to show $\frac{1}{4}$. How many should we make red?
- Students** 1.
- Teacher** What fraction?
- Students** $\frac{1}{4}$.
- Teacher** So, we have $\frac{7}{8}$ minus $\frac{1}{4}$. Let's subtract by separating. What does separating mean?
- Students** To take away.

Teacher Yes. Let's separate, or take away, the parts of the fractions. When subtracting fractions, first we want to determine whether the denominators are like or unlike. You might also say common or uncommon denominators. Are the denominators the same or alike?

Students No.

Teacher How do you know the denominators are not alike?

Students We have a denominator of 8 and a denominator of 4. Those are not the same.

Teacher The denominators are not the same. To subtract, we should subtract the parts of the subtrahend from the parts of the minuend. When the denominators are unlike, the parts or numerators do not represent the same value. So, we will work to make the fractions have like denominators. Why do we want to subtract fractions with like denominators?

Students So we can subtract the parts or numerators of the fractions.

Teacher To do this, let's write the first five multiples of each denominator. The minuend has a denominator of 8, so let's write the first five multiples of 8. (Write multiples as 8, 16, 24, 32, 40.)

Teacher What are the multiples of 8? Say them with me.

Students 8, 16, 24, 32, 40.

Teacher The subtrahend has a denominator of 4, so let's write the first five multiples of 4. (Write multiples as 4, 8, 12, 16, 20.)

Teacher What are the multiples of 4? Say them with me.

Students 4, 8, 12, 16, 20.

Teacher Great. Let's determine the least common multiple of the two fractions. What is the multiple with the least value that you see on both lists of multiples?

Students 8.

Teacher So, 8 is the least common multiple. What is 8?

Students The least common multiple.

Teacher Sometimes we call the least common multiple the LCM. What do we call the least common multiple?

Students LCM.

Teacher The least common multiple, or LCM, helps us determine the common denominator for the two fractions. What does the LCM help with?

Students Finding a common denominator for the two fractions.

Teacher The minuend has a denominator of 8, which is the original denominator. We don't need to convert this fraction. What do we need to do?

Students Nothing.

Teacher What's the subtrahend?

Students $\frac{1}{4}$.

Teacher The subtrahend has a denominator of 4, which is not the original denominator. We need to convert the fraction from a denominator of 4 to a denominator of 8. What do we need to do?

Students Convert the fraction from a denominator of 4 to a denominator of 8.

Teacher To convert the fraction to a denominator of 8, I determine how many groups of 4 I need to make 8. I see I need to make 1, 2 groups of 4. (Point to the multiples of 4 and 8.) **How many groups?**

Students 2.

Teacher Let's make 2 groups of the fraction $\frac{1}{4}$ with the two-color counters. We already have one group of $\frac{1}{4}$. Let's make a second group (show 1 red counter and 3 yellow counters). Our new fraction is $\frac{2}{8}$. Is $\frac{2}{8}$ equivalent to $\frac{1}{4}$?

Students Yes. The fractions are equivalent.

Teacher Now that we have common denominators, we want to subtract the subtrahend parts or numerator from the minuend parts or numerator. That means we need to subtract 2 one-eighth parts from 7 one-eighth parts. **What do we subtract?**

Students We subtract the parts or numerators of the fractions.

Teacher Let's subtract the parts or numerators. With the two-color counters, we leave the subtrahend set alone. We subtract the 2 one-eighth parts by turning over the parts or numerators of the minuend. **How many parts do we subtract?**

Students 2.

Teacher We subtract the 2 one-eighth parts. We now have 1, 2, 3, 4, 5 one-eighth parts. **How many parts?**

Students 5.

Teacher When you have $\frac{7}{8}$ minus $\frac{2}{8}$, the difference is $\frac{5}{8}$. **What's the difference?**

Students $\frac{5}{8}$.

Teacher $\frac{7}{8}$ minus $\frac{2}{8}$ equals $\frac{5}{8}$. **Let's say that together.**

Students $\frac{7}{8}$ minus $\frac{2}{8}$ equals $\frac{5}{8}$.

Teacher If you have a set of $\frac{7}{8}$ and subtract a set of $\frac{1}{4}$, the difference is $\frac{5}{8}$. $\frac{7}{8}$ minus $\frac{2}{8}$ equals $\frac{5}{8}$. **Let's review. What's a minuend?**

Students The number from which another is subtracted.

Teacher **What's a subtrahend?**

Students The number to be subtracted.

Teacher **What's a difference?**

Students The amount between the minuend and subtrahend.

Teacher **What does it mean to separate?**

Students To take away.

Teacher **How could you explain separating to a friend?**

Students We showed the minuend and subtrahend. We used the LCM to determine the common denominator. Then, we subtracted the parts of the subtrahend from the parts of the minuend to learn the difference.

Teacher **What's another way we could have solved this problem?**

Students We could have compared two sets.

(3) Subtraction of Decimals with Traditional Algorithm

Routine

Materials:

- [Module 9 Problem Sets](#)
- [Module 9 Vocabulary Cards](#)
 - If necessary, review Vocabulary Cards before teaching
- A hands-on tool or manipulative like Base-10 blocks or money
 - Note that drawings can be used alongside or instead of manipulatives

ROUTINE WITH MANIPULATIVES

Teacher	Let's work on subtraction. What does it mean to subtract?
Students	To separate or compare.
Teacher	Subtraction means to separate from a set or to compare two sets. Look at this problem. (Show problem.)
Teacher	First, I see a minus sign (point). The minus sign tells us to subtract. What does the minus sign mean?
Students	To subtract.
Teacher	Let's do this problem with our number line. (Show number line.)
Teacher	When we use the Base-10 blocks with decimals, we can shift the meaning of each type of block. Today, let's use the flats to represent ones. What do the flats represent?
Students	Ones.
Teacher	We'll use the rods to represent tenths. What do the rods represent?
Students	Tenths.
Teacher	How can we use the rods to represent tenths?
Students	1 rod equals 1 tenth.
Teacher	What do you notice about the relationship between the rods and the flat?
Students	There are 10 tenths in 1 in the same way there are 10 rods in 1 flat.
Teacher	With our Base-10 blocks, the units represent hundredths. What do the units represent?
Students	Hundredths.
Teacher	What do you notice about the relationship between the units and the rods?
Students	There are 10 hundredths in 1 tenth in the same way there are 10 units in 1 rod.
Teacher	Our minuend is __. What's our minuend?
Students	__.
Teacher	Let's show the minuend by showing __ ones, __ tenths, and __ hundredths. (Show with Base-10 blocks.)
Teacher	How many?
Students	__.
Teacher	Our subtrahend is __. What's our subtrahend?

Students ___.

Teacher **Let's show the subtrahend by showing ___ ones, ___ tenths, and ___ hundredths. (Show with Base-10 blocks.)**

Teacher **How many?**

Students ___.

Teacher **So, we have ___ minus ___. Let's subtract by separating. What does separating mean?**

Students To take away.

Teacher **Yes. Let's separate or take away. First, let's subtract the least place value. That means the place value with the least or smallest value. What's the least place value in this problem?**

Students Hundredths.

Teacher **Let's subtract the hundredths.**
(Subtract the subtrahend hundredths from the minuend hundredths.)

Teacher **Let's separate ___ hundredths from ___ hundredths. Do we have enough minuend hundredths to separate the ___ subtrahend hundredths?**

Students Yes.

Teacher **If we don't have enough hundredths, we have to regroup. Do we have to regroup?**

Students No.

Teacher **So, let's separate the subtrahend hundredths from the minuend hundredths. (Remove hundredths.)**

Teacher **How many hundredths are remaining?**

Students ___.

Teacher **Yes! There are ___ hundredths remaining. We leave the remaining hundredths here. Now, let's subtract the tenths. What should we subtract?**

Students The tenths.

Teacher **Let's separate ___ tenths from ___ tenths. Do we have enough minuend tenths to separate the ___ subtrahend tenths?**

Students No.

Teacher **That means we have to regroup. To regroup, we exchange 1 one for 10 tenths. How do we regroup?**

Students We exchange 1 one for 10 tenths.
(Show regrouping.)

Teacher **Now, we have ___ tenths and can subtract ___ tenths. Let's separate the subtrahend tenths from the minuend tenths.**
(Remove tenths.)

Teacher **How many tenths are remaining?**

Students ___.

Teacher **There are ___ tenths remaining. We leave the remaining tenths here. Now, let's subtract the ones. What should we subtract?**

Students The ones.

Teacher **Let's separate ___ ones from ___ ones. Do we have enough minuend ones to separate the subtrahend ones?**

Students Yes.

Teacher **We don't have to regroup. Let's subtract the ones.**
(Remove ones.)

Teacher **How many ones are remaining?**

Students ___.

Teacher **So, let's count the ones, tenths, and hundredths to learn the difference.**
Ready?
(Count the ones, then tenths, then hundredths.)

Teacher **That means ___ minus ___ equals ___. Let's say that together.**

Students ___ minus ___ equals ___.

Teacher **Let's say it together again.**

Students ___ minus ___ equals ___.

Teacher **So, if you have a set of ___ and subtract a set of ___, the difference is ___. ___ minus ___ equals ___. Let's review. What's a minuend?**

Students The number from which another is subtracted.

Teacher **What's a subtrahend?**

Students The number to be subtracted.

Teacher **What's a difference?**

Students The amount between the minuend and subtrahend.

Teacher **What does it mean to separate?**

Students To take away.

Teacher **How could you explain separating to a friend?**

Students We subtracted the hundredths, then the tenths, then the ones to learn the difference.

Teacher **What's another way we could have solved this problem?**

Students We could have compared two sets.

ROUTINE WITHOUT MANIPULATIVES

Teacher **Let's work on subtraction. What does it mean to subtract?**

Students To separate or compare.

Teacher **Subtraction means to separate from a set or compare two sets. Look at this problem.**
(Show problem.)

Teacher **First, I see a minus sign (point). The minus sign tells us to subtract. What does the minus sign mean?**

Students To subtract.

Teacher **Let's do this problem with our pencil. First, when I see a problem like this that requires computation, I like to draw vertical lines to separate the different place value columns. Let's draw a vertical line between the ones column and the tenths column and another vertical line between the tenths column and the hundredths column.**
(Draw vertical lines to separate place value columns.)

Teacher **Now, we start by subtracting the hundredths. What should we subtract first?**

Students The hundredths.

Teacher **Which hundredths do we subtract?**

Students ___ minus ___.

Teacher **Do you have enough minuend hundredths to subtract?**

Students No.

Teacher **So, we have to regroup. To regroup, we regroup/trade/exchange 1 tenth for 10 hundredths. I subtract 1 tenth from the tenths column. ___ minus 1 equals ___. I like to cross out the ___ and write a ___ in the tenths column.**
(Show subtraction of 1 tenth.)

Teacher **Now, I imagine regrouping this 1 tenth into 10 hundredths. If I have 10 hundredths and add these hundredths to the ___ hundredths, how many hundredths do I have now?**

Students ___.

Teacher **I like to show the ___ hundredths by crossing out the ___ and writing ___ in the hundredths column.**
(Show addition of 10 hundredths.)

Teacher **Now, let's subtract the hundredths. What's ___ minus ___?**
(If a student has difficulty with subtraction, say: **Start with the subtrahend. Place that number in your fist, and let's count up to the minuend. Ready? ___:** ___, ___, ___. See Counting Up poster at the end of Module 7 for more information.)

Students ___.

Teacher **Great. There are ___ hundredths. Let's write ___ below the equal line.**
(Write hundredths.)

Teacher **Now, let's subtract the tenths. Which tenths do we subtract?**

Students ___ minus ___.

Teacher **Do you have enough tenths to subtract ___ tenths?**

Students Yes.

Teacher **You have enough tenths to subtract or take away ___ tenths. We don't have to regroup. What's ___ minus ___?**

Students ___.

Teacher **There are ___ tenths. Let's write ___ below the equal line.**
(Write tenths.)

Teacher **Now, let's subtract the ones. Which ones do we subtract?**

Students ___ minus ___.

Teacher **Do you have enough ones to subtract ___ ones?**

Students Yes.

Teacher **You have enough ones to subtract. You don't have to regroup. What's ___ minus ___?**

Students ___.

Teacher **Let's write ___ below the equal line.**

Students ___.

Teacher **So, let's look at the problem. What's ___ minus ___?**

Students ___.

Teacher That's right. $__$ minus $__$ equals $__$. Let's say that together.
 Students $__$ minus $__$ equals $__$.
 Teacher So, if you have a set of $__$ and subtract a set of $__$, the difference is $__$. $__$ minus $__$ equals $__$. Let's review. What's a minuend?
 Students The number from which another is subtracted.
 Teacher What's a subtrahend?
 Students The number to be subtracted.
 Teacher What's a difference?
 Students The result of subtracting a subtrahend from a minuend.
 Teacher What does it mean to separate?
 Students To take away.
 Teacher How could you explain separating to a friend?
 Students We subtracted the hundredths but we didn't have enough hundredths so we regrouped 1 tenth for 10 hundredths. Then, we subtracted the tenths. Then, we subtracted the ones. We figured out the difference between $__$ and $__$.
 Teacher What's another way we could have solved this problem?
 Students We could have compared two sets.

Example

3.25
– 2.89

0.36

EXAMPLE WITH MANIPULATIVES

Teacher Let's work on subtraction. What does it mean to subtract?
 Students To separate or compare.
 Teacher Subtraction means to separate from a set or compare two sets. Look at this problem.
 (Show problem.)
 Teacher First, I see a minus sign (point). The minus sign tells us to subtract. What does the minus sign mean?
 Students To subtract.
 Teacher Let's do this problem with Base-10 blocks.
 (Move Base-10 blocks to workspace.)
 Teacher When we use the Base-10 blocks with decimals, we can shift the meaning of each type of block. Today, let's use the flats to represent ones. What do the flats represent?
 Students Ones.
 Teacher We'll use the rods to represent tenths. What do the rods represent?
 Students Tenths.
 Teacher How can we use the rods to represent tenths? What do you notice about the relationship between the rods and the flat?

Students There are 10 tenths in 1 in the same way there are 10 rods in 1 flat.
Teacher With our Base-10 blocks, the units represent hundredths. What do the units represent?

Students Hundredths.
Teacher What do you notice about the relationship between the units and the rods?

Students There are 10 hundredths in 1 tenth in the same way there are 10 units in 1 rod.
Teacher Our minuend is 3 and 25 hundredths. What's our minuend?

Students 3 and 25 hundredths.
Teacher Let's show the minuend by showing 3 ones, 2 tenths, and 5 hundredths. (Show with Base-10 blocks.)

Teacher How many?

Students 3 and 25 hundredths.
Teacher Our subtrahend is 2 and 89 hundredths. What's our subtrahend?

Students 2 and 89 hundredths.
Teacher Instead of showing the subtrahend, let's subtract the subtrahend from the minuend. What should we do?

Students Subtract the subtrahend from the minuend.
Teacher Let's start by subtracting the least place value. What's the least place value in this problem?

Students Hundredths.
Teacher How many hundredths do we subtract?

Students 9.
Teacher We need to subtract 9 hundredths. How many hundredths are in the minuend?

Students 5.
Teacher Do you have enough hundredths to subtract 9 hundredths?

Students No.
Teacher So, let's regroup. Let's regroup/trade/exchange 1 tenths for 10 hundredths. (Show 1 tenth as equivalent to 10 hundredths.)

Teacher I place the 10 hundredths in the hundredths column. (Place 10 hundredths in hundredths column.)

Teacher Now we have 15 hundredths. How many hundredths?

Students 15.
Teacher Let's subtract the hundredths. That means we subtract 9 hundredths from 15 hundredths. (Move hundredths.)

Teacher How many hundredths remaining?

Students 6 hundredths.
Teacher Let's subtract the tenths. We have 1 tenth in the minuend and we need to subtract 8 tenths of the subtrahend. Do we have enough tenths to subtract 8 tenths?

Students No.
Teacher What do we have to do?

Students Regroup.

Teacher We need to regroup 1 one for 10 tenths. Let's regroup/trade/exchange 1 one for 10 tenths.
(Show 1 one as equivalent to 10 tenths.)

Teacher I place the 10 tenths in the tenths column.
(Place 10 tenths in the tenths column.)

Teacher Now we have 11 tenths. How many tenths?
Students 11.

Teacher Let's subtract the tenths. We subtract 8 tenths from 11 tenths.
(Move tenths.)

Teacher How many tenths remaining?
Students 3 tenths.

Teacher Now, let's subtract the ones. We have 2 ones in the minuend and 2 ones in the subtrahend. Do we have enough ones to subtract?
Students Yes.

Teacher What's 2 minus 2?
Students 0.

Teacher So, let's count the ones, tenths, and hundredths to learn the difference. Ready?
(Count the ones, then tenths, then hundredths.)

Teacher That means 3 and 25 hundredths minus 2 and 89 hundredths equals 36 hundredths. Let's say that together.
Students 3 and 25 hundredths minus 2 and 89 hundredths equals 36 hundredths.

Teacher Let's say it together again.
Students 3 and 25 hundredths minus 2 and 89 hundredths equals 36 hundredths.

Teacher Let's review. What's a minuend?
Students The number from which another is subtracted.

Teacher What's a subtrahend?
Students The number to be subtracted.

Teacher What's a difference?
Students The amount between the minuend and subtrahend.

Teacher What does it mean to separate?
Students To take away.

Teacher How could you explain separating to a friend?
Students We subtracted the hundredths but first we had to regroup. Then, we subtracted the tenths but we also had to regroup. Then, we subtracted the ones. The difference between 3 and 25 hundredths and 2 and 89 hundredths is 36 hundredths.

Teacher What's another way we could have solved this problem?
Students We could have compared two sets.

(4) Subtraction of Decimals with Adding Up Algorithm

Routine

Materials:

- [Module 9 Problem Sets](#)
- [Module 9 Vocabulary Cards](#)
 - If necessary, review Vocabulary Cards before teaching
- A hands-on tool or manipulative like Base-10 blocks or money
 - Note that drawings can be used alongside or instead of manipulatives

ROUTINE WITH MANIPULATIVES

- Teacher** Let's work on subtraction. What does it mean to subtract?
- Students** To separate or compare.
- Teacher** Subtraction means to separate from a set or to compare two sets. Look at this problem.
(Show problem.)
- Teacher** First, I see a minus sign (point). The minus sign tells us to subtract. What does the minus sign mean?
- Students** To subtract.
- Teacher** Today, let's think about subtraction as the difference between two numbers. How can we interpret subtraction?
- Students** The difference between two numbers.
- Teacher** So, in this problem, subtraction is the difference between what two numbers?
- Students** ___ and ___.
- Teacher** Let's figure out the difference between ___ and ___. Let's do this with our Base-10 blocks.
(Show Base-10 blocks.)
- Teacher** When we think about subtraction as the difference between two numbers, let's start with our subtrahend. What's the subtrahend in this problem?
- Students** ___.
- Teacher** Let's show the subtrahend with our Base-10 blocks. How many ones?
- Students** ___.
- Teacher** How many tenths?
- Students** ___.
- Teacher** How many hundredths?
- Students** ___.
(Show subtrahend with Base-10 blocks.)
- Teacher** Now, let's think about what we could add to the subtrahend to reach the minuend, ___. I see that I could add ___ hundredths to get to the nearest tenth. I'll add the hundredths over here so I don't confuse these hundredths with the subtrahend hundredths.
(Add hundredths in separate pile.)

Teacher Now, what else could we add to reach the minuend, ___? I see that I could add ___ tenths to get very close to the minuend of ___. I'll add the tenths over here so I don't confuse these tenths with the subtrahend tenths.
(Add tenths.)

Teacher Have we reached the minuend yet?

Students No.

Teacher What could we add to reach the minuend?

Students ___.

Teacher I could add ___ ones to reach the minuend. Let's add the ones over here so I don't confuse these ones with the subtrahend ones.
(Add ones.)

Teacher So, the difference between ___ and ___ is: __, __, __ ... What's the difference?

Students ___.

Teacher That means ___ minus ___ equals ___. Let's say that together.

Students ___ minus ___ equals ___.

Teacher Let's say it together again.

Students ___ minus ___ equals ___.

Teacher With this strategy, called adding up, you figure out the difference between ___ and ___ by adding up. You add up to find the difference between ___ and ___.
How do you find the difference?

Students Adding up from ___ to ___.

Teacher Let's review. What's a minuend?

Students The number from which another is subtracted.

Teacher What's a subtrahend?

Students The number to be subtracted.

Teacher What's a difference?

Students The result of subtracting a subtrahend from a minuend.

Teacher How could you explain adding up to a friend?

Students You start with the subtrahend. You keep adding until you reach the minuend. You add up to find the difference between the minuend and subtrahend.

Example

5.17	2.99		
$- \underline{2.99}$	3.00	$+ .01$	
	5.00	$+ 2.00$	
	5.17	$+ .17$	
		$\underline{\quad}$	
		2.18	

EXAMPLE WITHOUT MANIPULATIVES

- Teacher** Let's work on subtraction. What does it mean to subtract?
- Students** To separate or compare.
- Teacher** Subtraction means to separate from a set or to compare two sets. Look at this problem.
(Show problem.)
- Teacher** First, I see a minus sign (point). The minus sign tells us to subtract. What does the minus sign mean?
- Students** To subtract.
- Teacher** Today, let's think about subtraction as the difference between two numbers. How can we interpret subtraction?
- Students** The difference between two numbers.
- Teacher** So, in this problem, subtraction is the difference between what two numbers?
- Students** 5.17 and 2.99.
- Teacher** Let's figure out the difference between 5.17 and 2.99.
- Teacher** When we think about subtraction as the difference between two numbers, let's start with our subtrahend. What's the subtrahend in this problem?
- Students** 2.99.
- Teacher** Let's write the subtrahend next to the problem. What should we write?
- Students** 2.99.
- Teacher** Now, let's think about what we could add to 2.99 to reach the minuend, 5.17. I see that I could add 1 hundredth to get to the nearest tenth. I'll write +.01 over here to show I wanted to add 1 hundredth.
(Write +.01.)
- Teacher** If I added .01 to 2.99, what's the sum?
- Students** 3.00.
- Teacher** Let's write 3.00 below 2.99 to remember we're now at 3.00.
(Write 3.00 below 2.99.)
- Teacher** Let's figure out what we could add to 3.00 to reach the minuend, 5.17. Could we add 2 more to get to 5?
- Students** Yes.
- Teacher** Let's write +2.00 to show we wanted to add 2 ones.
(Write +2.00 below +.01.)
- Teacher** If we added 2 to 3, what's the sum?
- Students** 5.
- Teacher** Let's write 5.00 below 3.00 to remember we're now at 5.00.
(Write 5.00 below 3.00.)
- Teacher** Let's keep going. What could we add to 5.00 to reach the minuend?
- Students** .17.
- Teacher** Great idea. Let's write +.17 to show we wanted to add .17.
(Write +.17.)
- Teacher** If I added .17 to 5.00, what's the sum?
- Students** 5.17.
- Teacher** Let's write 5.17 below 5.00 to remember we're now at 5.17.

(Write 5.17 below 5.00.)

Teacher Did we reach the minuend?
Students Yes!
Teacher Now, we add **+0.01** and **+2.00** and **+0.17** to determine the difference. How could we add these numbers?
Students $2.00 + .17 + .01$ (or other responses).
Teacher So, the difference is **2.18**. What's the difference?
Students 2.18.
Teacher That means **5.17** minus **2.99** equals **2.18**. Let's say that together.
Students 5.17 minus 2.99 equals 2.18.
Teacher Let's say it together again.
Students 5.17 minus 2.99 equals 2.18.
Teacher With this strategy, called **adding up**, you figure out the difference between **5.17** and **2.99** by **adding up**. How do you find the difference?
Students Adding up from 2.99 to 5.17.
Teacher Let's review. What's a minuend?
Students The number from which another is subtracted.
Teacher What's a subtrahend?
Students The number to be subtracted.
Teacher What's a difference?
Students The result of subtracting a subtrahend from a minuend.
Teacher How could you explain adding up to a friend?
Students You start with the subtrahend. You keep adding until you reach the minuend. You add up to find the difference between the minuend and subtrahend.

D. Problems for Use During Instruction

[See Module 9 Problem Sets.](#)

E. Vocabulary Cards for Use During Instruction

[See Module 9 Vocabulary Cards.](#)

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Module 9:

Subtraction of Rational Numbers

Problem Sets

- A. Proper fractions with like denominators and sums <1 (20)
- B. Improper fractions with like denominators and sums >1 (10)
- C. Mixed numbers with like denominators and sums >1 (10)
- D. Proper fractions with unlike denominator and sums <1 (20)
- E. Improper fractions with unlike denominator and sums >1 (10)
- F. Mixed numbers with unlike denominator and sums >1 (10)

- G. Decimals with tenths; no regrouping (20)
- H. Decimals with tenths; regrouping (20)
- I. Decimals with hundredths; no regrouping (20)
- J. Decimals with hundredths; regrouping (20)
- K. Decimals with tenths and hundredths; mix of regrouping (20)

A.

$$\frac{4}{5} - \frac{1}{5} =$$

A.

$$\frac{6}{10} - \frac{3}{10} =$$

A.

$$\frac{3}{6} - \frac{1}{6} =$$

A.

$$\frac{2}{4} - \frac{1}{4} =$$

A.

$$\frac{2}{3} - \frac{1}{3} =$$

A.

$$\frac{4}{6} - \frac{2}{6} =$$

A.

$$\frac{6}{8} - \frac{1}{8} =$$

A.

$$\frac{4}{10} - \frac{1}{10} =$$

A.

$$\frac{6}{12} - \frac{4}{12} =$$

A.

$$\frac{4}{10} - \frac{1}{10} =$$

A.

$$\frac{9}{12} - \frac{4}{12} =$$

A.

$$\frac{3}{5} - \frac{1}{5} =$$

A.

$$\frac{5}{6} - \frac{1}{6} =$$

A.

$$\frac{4}{7} - \frac{1}{7} =$$

A.

$$\frac{6}{9} - \frac{2}{9} =$$

A.

$$\frac{6}{10} - \frac{1}{10} =$$

A.

$$\frac{3}{4} - \frac{1}{4} =$$

A.

$$\frac{5}{6} - \frac{3}{6} =$$

A.

$$\frac{6}{7} - \frac{3}{7} =$$

A.

$$\frac{6}{8} - \frac{2}{8} =$$

B.

$$\frac{10}{5} - \frac{4}{5} =$$

B.

$$\frac{16}{12} - \frac{3}{12} =$$

B.

$$\frac{11}{6} - \frac{3}{6} =$$

B.

$$\frac{8}{4} - \frac{1}{4} =$$

B.

$$\frac{8}{3} - \frac{2}{3} =$$

B.

$$\frac{9}{6} - \frac{1}{6} =$$

B.

$$\frac{14}{8} - \frac{5}{8} =$$

B.

$$\frac{13}{10} - \frac{1}{10} =$$

B.

$$\frac{17}{12} - \frac{4}{12} =$$

B.

$$\frac{18}{10} - \frac{4}{10} =$$

c.

$$7\frac{7}{8} - 4\frac{3}{8} =$$

c.

$$2\frac{3}{5} - 1\frac{3}{5} =$$

c.

$$2\frac{5}{6} - 1\frac{2}{6} =$$

c.

$$2\frac{6}{10} - 1\frac{1}{10} =$$

c.

$$2\frac{6}{9} - 1\frac{4}{9} =$$

c.

$$7\frac{3}{5} - 1\frac{4}{5} =$$

c.

$$7\frac{3}{4} - 4\frac{1}{4} =$$

c.

$$7\frac{4}{6} - 2\frac{4}{6} =$$

c.

$$2\frac{5}{8} - 1\frac{4}{8} =$$

c.

$$2\frac{6}{8} - 1\frac{5}{8} =$$

D.

$$\frac{3}{5} - \frac{1}{3} =$$

D.

$$\frac{5}{6} - \frac{1}{2} =$$

D.

$$\frac{2}{4} - \frac{1}{12} =$$

D.

$$\frac{3}{5} - \frac{1}{10} =$$

D.

$$\frac{1}{3} - \frac{1}{6} =$$

D.

$$\frac{2}{5} - \frac{1}{10} =$$

D.

$$\frac{4}{8} - \frac{2}{16} =$$

D.

$$\frac{3}{6} - \frac{1}{3} =$$

D.

$$\frac{2}{3} - \frac{1}{2} =$$

D.

$$\frac{6}{8} - \frac{2}{4} =$$

D.

$$\frac{9}{10} - \frac{2}{5} =$$

D.

$$\frac{12}{5} - \frac{3}{10} =$$

D.

$$\frac{2}{3} - \frac{2}{5} =$$

D.

$$\frac{1}{3} - \frac{1}{4} =$$

D.

$$\frac{2}{4} - \frac{2}{5} =$$

D.

$$\frac{1}{2} - \frac{1}{5} =$$

D.

$$\frac{2}{4} - \frac{2}{12} =$$

D.

$$\frac{7}{9} - \frac{1}{3} =$$

D.

$$\frac{3}{4} - \frac{5}{8} =$$

D.

$$\frac{2}{3} - \frac{2}{12} =$$

E.

$$\frac{15}{3} - \frac{4}{2} =$$

E.

$$\frac{5}{4} - \frac{1}{5} =$$

E.

$$\frac{17}{4} - \frac{5}{2} =$$

E.

$$\frac{16}{5} - \frac{5}{3} =$$

E.

$$\frac{11}{4} - \frac{10}{8} =$$

E.

$$\frac{19}{2} - \frac{8}{6} =$$

E.

$$\frac{7}{4} - \frac{5}{8} =$$

E.

$$\frac{30}{3} - \frac{1}{3} =$$

E.

$$\frac{13}{5} - \frac{2}{10} =$$

E.

$$\frac{19}{15} - \frac{4}{5} =$$

F.

$$2\frac{1}{3} - 1\frac{2}{9} =$$

F.

$$7\frac{1}{10} - 1\frac{7}{8} =$$

F.

$$7\frac{1}{2} - 3\frac{1}{5} =$$

F.

$$2\frac{9}{12} - 1\frac{2}{4} =$$

F.

$$7\frac{1}{3} - 2\frac{7}{12} =$$

F.

$$2\frac{4}{10} - 1\frac{2}{5} =$$

F.

$$7\frac{3}{6} - 2\frac{5}{12} =$$

F.

$$2\frac{2}{3} - 1\frac{1}{9} =$$

F.

$$7\frac{1}{2} - 5\frac{5}{6} =$$

F.

$$2\frac{5}{6} - 1\frac{5}{12} =$$

G.

$$\begin{array}{r} 9.9 \\ - 2.1 \\ \hline \end{array}$$

G.

$$\begin{array}{r} 8.8 \\ - 3.2 \\ \hline \end{array}$$

G.

$$\begin{array}{r} 6.7 \\ - 5.5 \\ \hline \end{array}$$

G.

5.3

- 4.1



G.

$$\begin{array}{r} 9.8 \\ - 5.2 \\ \hline \end{array}$$

G.

8.6

- 2.3



G.

$$\begin{array}{r} 0.3 \\ - 0.1 \\ \hline \end{array}$$

G.

2.2

- 1.2



G.

3.3

- 0.3

G.

4.5

- 2.2



G.

4.1

- 0.1

G.

$$\begin{array}{r} 9.6 \\ - 3.3 \\ \hline \end{array}$$

G.

2.8

- 0.1



G.

4.6

- 1.5



G.

6.3

- 2.1



G.

3.8

- 1.5



G.

5.7

- 4.1



G.

9.4

- 0.2

G.

6.7

- 1.2

G.

0.7

- 0.2



H.

8.4

- 2.9

H.

5.3

- 3.7



H.

8.5

- 4.8

H.

8.6

- 5.9



H.

6.5

- 4.6

H.

7.4

- 5.7

H.

8.3

- 5.6

H.

6.4

- 1.9

H.

9.5

- 4.6



H.

7.2

- 0.7

H.

5.5

- 3.6



H.

4.1

- 2.9

H.

8.6

- 4.7



H.

6.1

- 3.8



H.

8.6

- 1.8



H.

7.2

- 2.8

H.

6.5

- 0.6



H.

7.2

- 4.8



H.

8.2

- 2.9



H.

4.2

- 0.8



l.

0.73

- 0.21

l.

3.46

- 1.32

l.

6.58

- 2.11



l.

9.82

- 0.01

l.

8.34

- 0.22

l.

2.59

- 1.46

l.

2.61

- 1.30



l.

$$\begin{array}{r} 7.47 \\ - 2.31 \\ \hline \end{array}$$

l.

9.63

– 0.60

l.

12.46

- 1.10



1.

26.24

- 3.03



l.

4.71

- 1.10

1.

4.35

- 2.22

l.

$$\begin{array}{r} 1.88 \\ - 0.01 \\ \hline \end{array}$$

l.

3.63

- 1.21

I.

$$\begin{array}{r} 10.26 \\ - 10.13 \\ \hline \end{array}$$

1.

$$\begin{array}{r} 9.44 \\ - 2.34 \\ \hline \end{array}$$

l.

5.62

- 1.20

l.

6.48

- 4.01



I.

10.55

- 0.33



J.

3.56

- 2.47



J.

5.24

- 1.37



J.

5.45

- 3.78



J.

6.67

- 2.29



J.

2.14

- 1.47



J.

4.23

- 2.25



J.

4.71

- 3.89



J.

3.52

- 1.77



J.

6.84

- 2.16



J.

14.80

- 6.96



J.

7.83

- 6.99



J.

9.75

- 8.80



J.

$$\begin{array}{r} 46.80 \\ - 12.93 \\ \hline \end{array}$$

J.

3.14

- 1.99



J.

7.21

- 4.66



J.

5.44

- 2.08

J.

9.66

- 1.67

J.

8.33

- 1.92

J.

$$\begin{array}{r} 42.12 \\ - 10.09 \\ \hline \end{array}$$

J.

$$\begin{array}{r} 6.81 \\ - 2.33 \\ \hline \end{array}$$

K.

30.15

- 2.6



K.

2.5

- 1.49

K.

14.58

- 1.4



K.

$$\begin{array}{r} 10.2 \\ - 5.73 \\ \hline \end{array}$$

K.

5.4

— .54

K.

8.3

- .91

K.

4.6

— .64

K.

9.38

- .19



K.

10.21

- 5.6



K.

3.9

- 1.01



K.

$$\begin{array}{r} 17.72 \\ - 12.58 \\ \hline \end{array}$$

K.

$$\begin{array}{r} 42.1 \\ - 17.96 \\ \hline \end{array}$$

K.

$$\begin{array}{r} 9.3 \\ - 6.31 \\ \hline \end{array}$$

K.

9.0

- 8.12

K.

$$\begin{array}{r} 9.17 \\ - 2.7 \\ \hline \end{array}$$

K.

3.46

- 1.6



K.

9.9

- 4.23

K.

$$\begin{array}{r} 15.5 \\ - 12.22 \\ \hline \end{array}$$

K.

17.5

- 8.83

K.

$$\begin{array}{r} 9.2 \\ - 6.75 \\ \hline \end{array}$$

Module 9:

Subtraction of Rational Numbers

Vocabulary Cards

algorithm

compare

computation

decimal

denominator

difference

equal sign

equivalent

fraction

hundredths

improper fraction

least common multiple

minuend

minus sign

mixed number

multiple

numerator

ones

regroup/trade/exchange

separate

subtract/subtraction

subtrahend

tenths

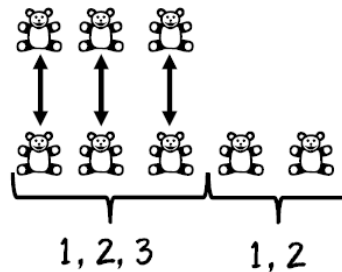
algorithm

A procedure or description of steps that can be used to solve a problem.

compare

To find the difference between two sets.

$$5 - 3 = 2$$



computation

The action used to solve a problem.

decimal

A number based on powers of ten.

34.107

tens ones tenths hundredths thousandths

denominator

The term in a fraction that tells the number of equal parts in a whole.

$$2 / 3 \quad \frac{2}{3} \quad \text{In these fractions, } 3 \text{ is the denominator.}$$

difference

The result of subtracting one number from another number.

$$6 - 4 = 2$$

2 is the **difference**

equal sign

The symbol that tells you that two sides of an equation are the same, balanced, or equal.

$$12 - 8 = 4$$

= is the equal sign

equivalent

Two numbers that have the same value.

$$\frac{1}{4} = \frac{2}{8} \qquad \frac{2}{3} = \frac{8}{12}$$

fraction

A number representing part of a whole or set.

$$\frac{3}{6} \quad \frac{10}{12} \quad \frac{8}{3}$$

hundredths

The digit in representing $\frac{1}{100}$.

In the number 4.23, 3 is in the hundredths place.

improper fraction

Any fraction in which the numerator is greater than the denominator.

$$\frac{9}{4} \quad \frac{17}{12} \quad \frac{10}{3}$$

least common multiple

The common multiple with the least value.

6: 6, 12, 18, 24, 30

8: 8, 16, 24, 32, 40

With multiples of 6 and 8, the **least common multiple** is 24.

minuend

The number from which another number is subtracted.

$$9 - 4 = 5$$

9 is the **minuend**

minus sign

The symbol that tells you to subtract.

$$9 - 4 = 5$$

- is the **minus sign**

mixed number

A whole number and a fraction combined.

$$1\frac{1}{6} \quad 4\frac{5}{12} \quad 12\frac{4}{3}$$

multiple

The product of a number and any integer.

4: 4, 8, 12, 16, 20

numerator

The term in a fraction that tells how many parts of a fraction.

$2/3$ $\frac{2}{3}$ In these fractions, **2** is the numerator.

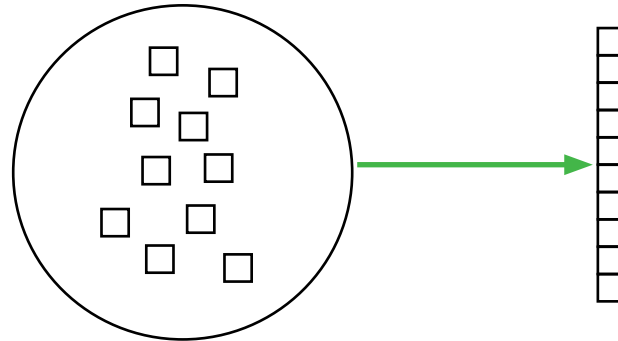
ones

The digit representing 1.

In the number **4.23**, **4** is in the ones place.

regroup/trade/exchange

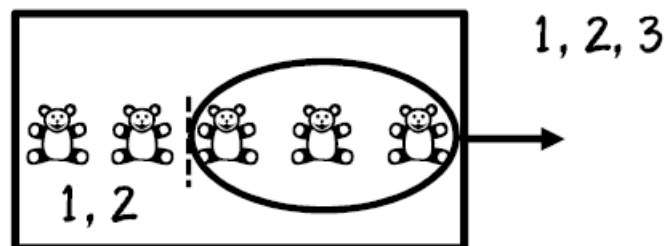
The process of exchanging 10 ones for 1 ten, 10 tens for 1 hundred, 10 hundreds for 1 thousand, etc.



separate

To start with a set and take away from that set.

$$5 - 3 = 2$$

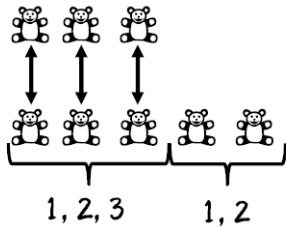


subtract/subtraction

To compare two sets or to separate from a set.

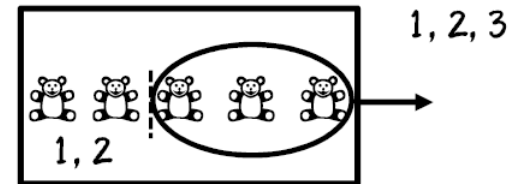
To compare two sets

$$5 - 3 = 2$$



To separate from a set

$$5 - 3 = 2$$



subtrahend

The number to be subtracted.

$$9 - 4 = 5$$

4 is the **subtrahend**

tenths

The digit in representing $\frac{1}{10}$.

In the number 4.23, 2 is in the tenths place.
